

MATH 152, Spring 2022
EXAM III - VERSION **A**

LAST NAME(print): _____ FIRST NAME(print): _____

UIN: _____

INSTRUCTOR: _____

SECTION NUMBER: _____

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and ness of the work leading up to it.
5. Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: _____

PART I: Multiple Choice. 4 points each.

1. Which of the following statements is true for the three series given below?

$$(I) \sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^3} \quad (II) \sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n} \quad (III) \sum_{n=2}^{\infty} \frac{(-1)^n (n)}{\ln n}$$

- (a) I converges absolutely, II and III converge conditionally.
- (b) I converges absolutely, II converges conditionally, and III diverges .
- (c) I and II converge conditionally, and III diverges.
- (d) I converges conditionally, II and III diverge.
- (e) I and II converge absolutely and III converges conditionally.

2. What is the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-2)^n (2n+1)!}{(n+1)!}$?

- (a) $\{2\}$
- (b) $(-\infty, \infty)$
- (c) $[1.5, 2.5]$
- (d) $\{0\}$
- (e) $[1.5, 2.5)$

3. If we find the third degree Taylor Polynomial for $f(x) = e^{-2x}$ centered at 4, what is the coefficient of $(x-4)^3$?

- (a) $-\frac{8}{3}e^{-8}$
- (b) $-\frac{4}{3}e^{-8}$
- (c) $\frac{2}{3}e^{-8}$
- (d) $\frac{4}{3}e^{-8}$
- (e) $\frac{8}{3}e^{-8}$

4. If $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+1}}{5^n} (x-2)^n$, find $f^{(25)}(2)$, that is, the 25th derivative of $f(x)$ evaluated at $x = 2$.

(a) $f^{(25)}(2) = \frac{3^{(26)}(25)!}{5^{(25)}}$

(b) $f^{(25)}(2) = \frac{(-1)3^{(25)}}{5^{(25)}(25)!}$

(c) $f^{(25)}(2) = \frac{(-1)3^{(26)}}{5^{(26)}(25)!}$

(d) $f^{(25)}(2) = \frac{3^{(25)}(25)!}{5^{(26)}}$

(e) $f^{(25)}(2) = \frac{(-1)3^{(26)}(25)!}{5^{(25)}}$

5. When we apply the Ratio Test to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^{2n}}{n^2 + 100}$, we find

(a) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{3}$

(b) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$

(c) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{9}$

(d) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 3$

(e) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 9$

6. Which of the following is a power series representation for $f(x) = \frac{x}{x^3 + 8}$?

(a) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{8^{n+1}}, |x| < 2$

(b) $f(x) = \sum_{n=0}^{\infty} \frac{x^{3n+3}}{8^{n+1}}, |x| < 2$

(c) $f(x) = \sum_{n=0}^{\infty} \frac{x^{3n+1}}{8^{n+1}}, |x| < \frac{1}{2}$

(d) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{8^{n+1}}, |x| < \frac{1}{2}$

(e) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+3}}{8^{n+1}}, |x| < 2$

7. Which of the following is a power series representation for $f(x) = \frac{1}{(1-3x)^2}$?

(a) $f(x) = \sum_{n=0}^{\infty} 3^n n x^{n-1}, |x| < \frac{1}{3}$

(b) $f(x) = -\sum_{n=0}^{\infty} 3^{n-1}(n+1)x^n, |x| < \frac{1}{3}$

(c) $f(x) = \sum_{n=0}^{\infty} 3^n(n+1)x^n, |x| < \frac{1}{3}$

(d) $f(x) = -\sum_{n=0}^{\infty} 3^n n x^{n-1}, |x| < \frac{1}{3}$

(e) $f(x) = \sum_{n=0}^{\infty} 3^{n-1}(n+1)x^n, |x| < \frac{1}{3}$

8. Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n (\pi)^{2n}}{n!}$.

(a) 0

(b) -1

(c) $\cos(3\pi^2)$

(d) $e^{-3\pi^2}$

(e) $e^{3\pi^2}$

9. Using The Alternating Series Estimation Theorem, what is the minimum number of terms needed to find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ to within $\frac{1}{165}$?

(a) $n = 6$

(b) $n = 5$

(c) $n = 4$

(d) $n = 3$

(e) $n = 7$

10. For which of the following series is the Ratio Test inconclusive?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n!}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4^n}$

(d) $\sum_{n=1}^{\infty} \frac{n! \sin(n)}{8^n}$

(e) $\sum_{n=1}^{\infty} \frac{\cos(n)}{n!}$

11. Given the series $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges at $x = 3$. Which of the following statements **must** be true?

(a) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ diverges at $x = 4$.

(b) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ diverges at $x = -8$.

(c) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges at $x = -5$.

(d) The series $\sum_{n=1}^{\infty} c_n(x+2)^n$ converges at $x = -7$.

(e) None of the above statements must be true.

12. Which of the following is the correct Maclaurin series for $f(x) = x \sin(5x)$?

(a) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1} x^{2n+1}}{(2n+1)!}$

(b) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+2} x^{2n+1}}{(2n+1)!}$

(c) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+1} x^{2n+2}}{(2n+1)!}$

(d) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^n x^{2n+2}}{(2n+1)!}$

(e) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n+2} x^{2n+2}}{(2n+1)!}$

13. The series $\sum_{n=1}^{\infty} \frac{\sin(n) + 5}{n^{3/2}}$

- (a) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{4}{n^{3/2}}$
- (b) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{5}{n^{3/2}}$
- (c) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$
- (d) Converges by the comparison test with $\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}$
- (e) None of the above

14. For what values of p does the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge?

- (a) Only if $p > 1$.
- (b) Only if $p \geq 0$.
- (c) Only if $p \geq 1$.
- (d) Only if $p > 0$.
- (e) The series converges for all p .

True or False. On your scantron, bubble 'a' if true and bubble 'b' if false. One point each.

15. If $0 \leq a_n \leq b_n$ for every positive integer n and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ also converges.

16. If $0 \leq a_n \leq b_n$ for every positive integer n and $\lim_{n \rightarrow \infty} \left| \frac{b_{n+1}}{b_n} \right| < 1$, then $\sum_{n=1}^{\infty} a_n$ converges.

17. If $0 \leq a_n \leq b_n$ for every positive integer n and $\lim_{n \rightarrow \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} b_n$ diverges.

18. If $0 \leq a_n \leq b_n$ for every positive integer n and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ converges.

PART II: Free Response: Show all work and box your final answer!

19. (12 pts) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+3)^n}{4^n(2n+1)}$. Be sure to test the endpoints of the interval for convergence.

20. Consider $f(x) = x^3 \cos\left(\frac{x}{3}\right)$.

a.) (5 pts) Using the known Maclaurin series for $\cos x$, write $f(x) = x^3 \cos\left(\frac{x}{3}\right)$ as a Maclaurin series. Include the radius of convergence.

b.) (4 pts) Using the result above, evaluate $\int_0^{0.1} x^3 \cos\left(\frac{x}{3}\right) dx$ as a series.

21. **Using a comparison test or limit comparison test**, determine whether the series below converge or diverge. Show all work, as illustrated in class, by naming the test, applying the test, and drawing the correct conclusion.

(a) (4 pts) $\sum_{n=1}^{\infty} \frac{2^n}{n + 5^n}$

(b) (4 pts) $\sum_{n=3}^{\infty} \frac{1}{\sqrt[3]{4n^3 + 4n + 1}}$

(c) (4 pts) $\sum_{n=1}^{\infty} \frac{\cos n}{n^4 + n^2 + \sqrt{n}}$

22. (7 pts) Find the Taylor Series for $f(x) = \frac{1}{x^3}$ centered at 5. You do not need to find the radius or interval of convergence.

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Question	Points Awarded	Points
1-18		60
19		12
20		9
21		12
22		7
		100

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