

MATH 152, Fall 2022
COMMON EXAM I - VERSION **A**

LAST NAME(print): _____ FIRST NAME(print): _____

UIN: _____

INSTRUCTOR:  _____

SECTION NUMBER: _____

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: _____

This page intentionally left blank.

PART I: Multiple Choice. 4 points each

1. Compute $\int x^3 \sqrt{2+x^4} dx$

- (a) None of these
- (b) $\frac{1}{4} (2+x^4)^{3/2} + C$
- (c) $\frac{1}{6} (2+x^4)^{3/2} + C$
- (d) $\frac{8}{3} (2+x^4)^{3/2} + C$
- (e) $\frac{3}{8} (2+x^4)^{3/2} + C$

$$u = 2+x^4$$

$$du = 4x^3 dx$$

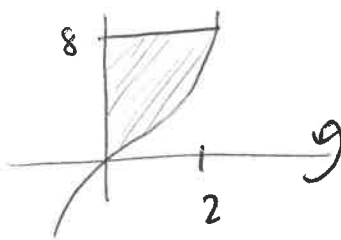
$$\frac{1}{4} du = x^3 dx$$

$$\int \frac{1}{4} u^{1/2} du = \frac{1}{4} \frac{2}{3} u^{3/2} + C$$

$$= \frac{1}{6} (2+x^4)^{3/2} + C$$

2. Consider the region bounded by the curves $y = x^3$, $y = 8$, and the y -axis. Which of the following represents the volume of this region being rotated about the x -axis?

- (a) $\int_0^8 \pi y^{2/3} dy$
- (b) $\int_0^2 \pi (8-x^3) dx$
- (c) None of these.
- (d) $\int_0^2 \pi (8-x^3)^2 dx$
- (e) $\int_0^2 \pi (64-x^6) dx$



$$r_o = 8$$

$$r_i = x^3$$

washer method.

$$V = \int_0^2 \pi (8^2 - (x^3)^2) dx$$

3. Suppose the work required to stretch a spring from its natural length to 4 m beyond its natural length is 16J. How much force is needed to hold the spring stretched 6 m beyond its natural length?

- (a) 24 N
- (b) 72 N
- (c) 12 N
- (d) 36 N
- (e) 18 N

$$W = 16 = \int_0^4 Kx dx = \left. \frac{K}{2} x^2 \right|_0^4 = \frac{16K}{2} = 8K$$

$$16 = 8K$$

$$2 = K$$

$$F = 2x$$

$$F(6) = 2(6) = 12$$

4. Which of the following represents the area bounded by the curves $y = x^2 - 2x$ and $y = 2x$ on the interval from $x = 1$ to $x = 6$?

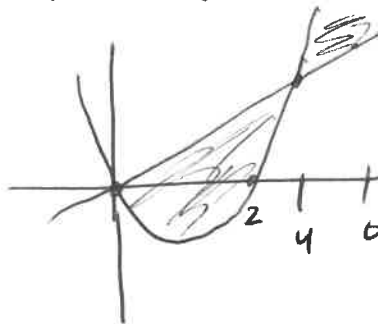
(a) $\int_0^6 4x - x^2 dx$

(b) $\int_0^4 4x - x^2 dx$

(c) $\int_1^4 x^2 - 4x dx + \int_4^6 4x - x^2 dx$

(d) $\int_1^6 4x - x^2 dx$

(e) $\int_1^4 4x - x^2 dx + \int_4^6 x^2 - 4x dx$



$$x^2 - 2x = 2x$$

$$x^2 - 4x = 0$$

$$x = 0 \quad x = 4$$

5. Compute $\int x^3 \sin(x) dx$

(a) $C - x^3 \cos(x) - 3x^2 \sin(x) + 6x \cos(x) + 6 \sin(x)$

(b) $C - x^3 \cos(x) + 3x^2 \sin(x) + 6x \cos(x) - 6 \sin(x)$

(c) $C + x^3 \cos(x) + 3x^2 \sin(x) - 6x \cos(x) + 6 \sin(x)$

(d) $C + x^3 \cos(x) + 3x^2 \cos(x) - 6x \sin(x) + 6 \cos(x)$

(e) $C + \frac{1}{4}x^4 - \frac{1}{3}x^3 \sin(x) + \frac{1}{6}x^2 \cos(x) + \frac{1}{6}x \sin(x)$

D	I
x^3	$\sin(x)$
$3x^2$	$-\cos(x)$
$6x$	$+\sin(x)$
6	$-\cos(x)$
0	$+\sin(x)$
\int	\int

$$= -x^3 \cos(x) + 3x^2 \sin(x) + 6x \cos(x) - 6 \sin(x) + \int 0 dx + C$$

6. Compute $\int_0^1 x^3(x^2 - 1)^4 dx$

- (a) $\frac{1}{60}$
- (b) $-\frac{1}{60}$
- (c) $\frac{1}{30}$
- (d) $-\frac{1}{30}$
- (e) $\frac{1}{15}$

$$u = x^2 - 1$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$x=0 \rightarrow u = -1$$

$$x=1 \rightarrow u = 0$$

$$x^2 = u + 1$$

$$\int_0^1 x^3(x^2 - 1)^4 dx = \int_{-1}^0 \frac{1}{2}(u+1)u^4 du$$

$$= \frac{1}{2} \int_{-1}^0 u^5 + u^4 du = \frac{1}{2} \left(\frac{u^6}{6} + \frac{u^5}{5} \right) \Big|_{-1}^0$$

$$= 0 - \frac{1}{2} \left(\frac{1}{6} + \frac{1}{5} \right) = -\frac{1}{2} \left(\frac{5}{30} + \frac{6}{30} \right)$$

$$= -\frac{1}{2} \left(-\frac{1}{30} \right) = \frac{1}{60}$$

7. Compute $\int \cos^4(x) \sin^5(x) dx = \int \cos^4(x) \sin^4(x) \sin(x) dx = \int \cos^4(x) (1 - \cos^2(x))^2 \sin(x) dx$

(a) $-\frac{1}{5} \cos^5(x) + \frac{1}{9} \cos^9(x) + C$

(b) $\frac{1}{6} \sin^6(x) - \frac{1}{4} \sin^8(x) + \frac{1}{10} \sin^{10}(x) + C$

(c) $\frac{1}{6} \sin^6(x) - \frac{1}{10} \sin^{10}(x) + C$

(d) $-\frac{1}{5} \cos^5(x) + \frac{2}{7} \cos^7(x) - \frac{1}{9} \cos^9(x) + C$

(e) None of these.

$u = \cos(x)$

$du = -\sin(x) dx$

$= \int -u^4 (1-u^2)^2 du$

$= \int -u^4 (1-2u^2+u^4) du$

$= \int -u^4 + 2u^6 - u^8 du$

$= -\frac{1}{5} u^5 + \frac{2}{7} u^7 - \frac{1}{9} u^9 + C$

8. Find the area bounded by $y = e^x$, $y = e^{-x}$, $x = 0$, and $x = 1$.

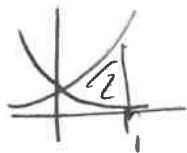
(a) $e + \frac{1}{e} - 2$

(b) $e - \frac{1}{e}$

(c) $e + \frac{1}{e} + 2$

(d) $1 + \frac{1}{e}$

(e) $1 + \frac{1}{e} - 2$



$\int_0^1 e^x - e^{-x} dx = e^x + e^{-x} \Big|_0^1$

$= e^1 + e^{-1} - (e^0 + e^0)$

$= e + \frac{1}{e} - 2$

9. Which of the following represents the volume of the region bounded by the curves $y = 6x - x^2$ and the x -axis being rotated about $x = -1$.

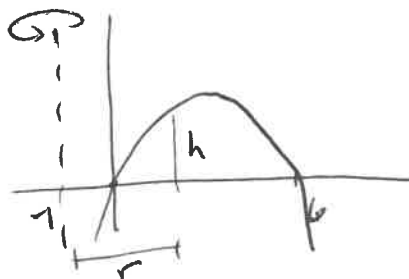
(a) $\int_0^6 2\pi x(6x - x^2) dx$

(b) $\int_0^6 2\pi(x-1)(6x - x^2) dx$

(c) $\int_0^6 \pi((6x - x^2)^2 - 1^2) dx$

(d) $\int_0^6 2\pi(1-x)(6x - x^2) dx$

(e) $\int_0^6 2\pi(x+1)(6x - x^2) dx$



Shell method

$h = 6x - x^2$

$r = x + 1$

10. Compute $\int \cos^3(2x) dx = \int \cos(2x) \cos^2(2x) dx = \int \cos(2x) (1 - \sin^2(2x)) dx$

(a) $-\sin(2x) + \frac{1}{3} \sin^3(2x) + C$

(b) $\frac{-1}{2} \sin(2x) + \frac{1}{6} \cos^3(2x) + C$

(c) None of these.

(d) $\sin(2x) - \frac{1}{3} \sin^3(2x) + C$

(e) $\frac{1}{2} \sin(2x) - \frac{1}{6} \sin^3(2x) + C$

$u = \sin(2x)$ $= \int \frac{1}{2} (1 - u^2) du$

$du = 2 \cos(2x) dx$

$\frac{1}{2} du = \cos(2x) dx$ $= \frac{1}{2} \left(u - \frac{u^3}{3} \right)$

$= \frac{1}{2} \sin(2x) - \frac{1}{6} \sin^3(2x) + C$

11. A cable, 20 feet long and weighing 6 pounds per foot, is hanging off the side of a 30 foot tall building. At the bottom of the cable is a bucket of rocks weighing 100 pounds. How much work is required to pull 10 feet of the cable to the top of the building?

(a) 900 ft-lbs

(b) 1900 ft-lbs

(c) 1300 ft-lbs

(d) 3200 ft-lbs

(e) 300 ft-lbs

Let $x = \#$ of ft of cable pulled up.

force = $120 - 6x + 100 = 220 - 6x$

$w = \int_0^{10} 220 - 6x dx = 220x - 3x^2 \Big|_0^{10}$

$= 2200 - 300 - (0)$

$= 1900$

12. Compute $\int \tan^3(x) \sec^3(x) dx$

(a) $\frac{1}{5} \sec^5(x) - \frac{1}{3} \sec^3(x) + C$

(b) $-\frac{1}{5} \sec^5(x) + \frac{1}{3} \sec^3(x) + C$

(c) $\frac{1}{5} \tan^5(x) - \frac{1}{3} \tan^3(x) + C$

(d) $-\frac{1}{5} \tan^5(x) + \frac{1}{3} \tan^3(x) + C$

(e) $-\sec^4(x) + \sec^2(x) + C$

$\int \tan^2 x \sec^2 x \tan x \sec x dx$

$\int (\sec^2 x - 1) \sec^2(x) \tan x \sec x dx$

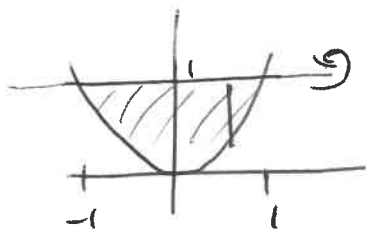
$u = \sec x$ $\int (u^2 - 1) u^2 du = \int u^4 - u^2 du$

$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C$

$= \frac{1}{5} \sec^5(x) - \frac{1}{3} \sec^3(x) + C$

13. The region bounded by the curves $y = x^2$ and $y = 1$ is rotated about the line $y = 1$. Find the volume of the resulting solid.

- (a) $\frac{8\pi}{15}$
 (b) $\frac{8\pi}{5}$
 (c) $\frac{4\pi}{3}$
 (d) $\frac{12\pi}{5}$
 (e) $\frac{16\pi}{15}$



disk method

$$r = 1 - x^2$$

$$V = \int_{-1}^1 \pi (1 - x^2)^2 dx$$

$$= 2 \int_0^1 \pi (1 - 2x^2 + x^4) dx = 2\pi \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]_0^1 = 2\pi \left(1 - \frac{2}{3} + \frac{1}{5} \right)$$

$$= 2\pi \left(\frac{15 - 10 + 3}{15} \right) = 2\pi \left(\frac{8}{15} \right) = \frac{16\pi}{15}$$

14. Compute $\int_1^e x^2 \ln x dx$.

- (a) $\frac{2}{9}e^3 - \frac{1}{9}$
 (b) $\frac{2}{9}e^3 + \frac{1}{9}$
 (c) $1 - e$
 (d) $e^2 - \frac{1}{9}e^3 + \frac{1}{9}$
 (e) None of these.

$$\int x^2 \ln(x) dx = \frac{x^3}{3} \ln(x) - \int \frac{1}{3} x^2 dx$$

$$= \frac{x^3}{3} \ln(x) - \frac{1}{9} x^3$$

$$\left(\frac{x^3}{3} \ln(x) - \frac{1}{9} x^3 \right) \Big|_1^e = \frac{e^3}{3} \ln(e) - \frac{1}{9} e^3 - \left(0 - \frac{1}{9} \right)$$

$$= \frac{e^3}{3} - \frac{1}{9} e^3 + \frac{1}{9}$$

$$= \frac{2}{9} e^3 + \frac{1}{9}$$

0	I
$\ln(x)$	$+ x^2$
$\frac{1}{x}$	$-\frac{x^3}{3}$

15. Compute $\int_0^{\pi/4} \sec^4(x) dx = \int_0^{\pi/4} \sec^2(x) \sec^2(x) dx$

(a) $\frac{2}{3}$

(b) $\frac{32}{5}$

(c) $\frac{4}{3}$

(d) $\frac{4\sqrt{2}}{5}$

(e) $\frac{2\sqrt{2}-1}{3}$

$$= \int_0^{\pi/4} (\tan^2 x + 1) \sec^2(x) dx = \int_0^1 u^2 + 1 du$$

$$u = \tan x \quad x=0 \rightarrow u=0$$

$$du = \sec^2 x \quad x=\frac{\pi}{4} \rightarrow u=1$$

$$= \left(\frac{u^3}{3} + u \right) \Big|_0^1$$

$$= \frac{1}{3} + 1 = \frac{4}{3}$$

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (6 points) Compute $\int \cos^2(x) \sin^2(x) dx$

$$\begin{aligned} &= \int \frac{1}{2}(1 + \cos(2x)) \cdot \frac{1}{2}(1 - \cos(2x)) dx \\ &= \frac{1}{4} \int 1 - \cos^2(2x) dx \\ &= \frac{1}{4} \int 1 - \frac{1}{2}[1 + \cos(4x)] dx \\ &= \frac{1}{4} \int 1 - \frac{1}{2} - \frac{1}{2}\cos(4x) dx \\ &= \frac{1}{4} \int \frac{1}{2} - \frac{1}{2}\cos(4x) dx \\ &= \frac{1}{4} \left[\frac{1}{2}x - \frac{1}{2} \cdot \frac{1}{4} \sin(4x) \right] + C \\ &= \frac{1}{4} \left[\frac{1}{2}x - \frac{1}{8} \sin(4x) \right] + C \end{aligned}$$

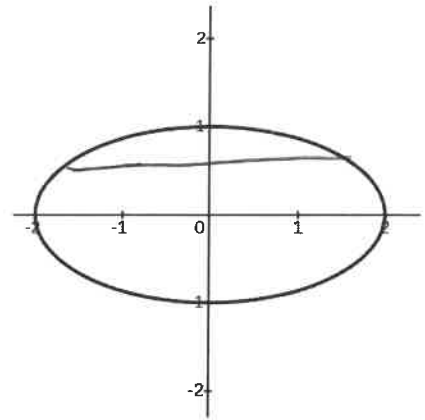
17. (8 points) Find the volume of the solid whose base is the ellipse $x^2 + 4y^2 = 4$ and whose cross-sections perpendicular to the y -axis are squares. Evaluate your integral.

$$\text{base} = 2x$$

$$A = b^2 = (2x)^2 = 4x^2$$

$$A = 4(4 - 4y^2)$$

$$A = 16 - 16y^2$$



$$x^2 + 4y^2 = 4$$

$$x^2 = 4 - 4y^2$$

$$V = \int A \, dy = \int_{-1}^1 16 - 16y^2 \, dy$$

$$= 2 \int_0^1 16 - 16y^2 \, dy = 2 \left(16y - \frac{16y^3}{3} \right) \Big|_0^1$$

$$= 2 \left[16 - \frac{16}{3} - (0) \right]$$

$$= 2 \left(\frac{48}{3} - \frac{16}{3} \right) = 2 \left(\frac{32}{3} \right) = \frac{64}{3}$$

18. Consider the region bounded by the curves $y = 7x - x^2$ and $y = 7 - x$

- (a) (8 points) **Set up** an integral to find the volume of the solid formed by rotating this region about the line $y = -2$. **Do not evaluate your integral.**

$$7x - x^2 = 7 - x$$

$$0 = x^2 - 8x + 7$$

$$(x-7)(x-1)$$

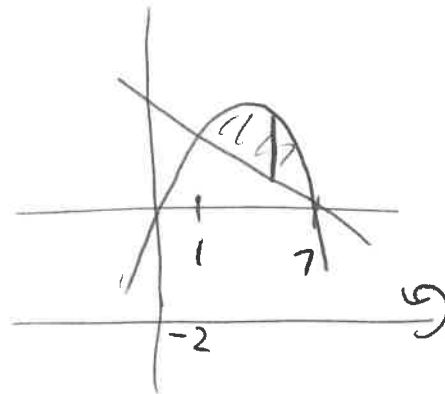
washer

$$r_o = 7x - x^2 - (-2)$$

$$= 7x - x^2 + 2$$

$$r_i = 7 - x - (-2)$$

$$= 7 - x + 2 = 9 - x$$



$$V = \int_1^7 \pi \left[(7x - x^2 + 2)^2 - (9 - x)^2 \right] dx$$

- (b) (8 points) **Set up** an integral to find the volume of the solid formed by rotating this region about the line $x = 10$. **Do not evaluate your integral.**

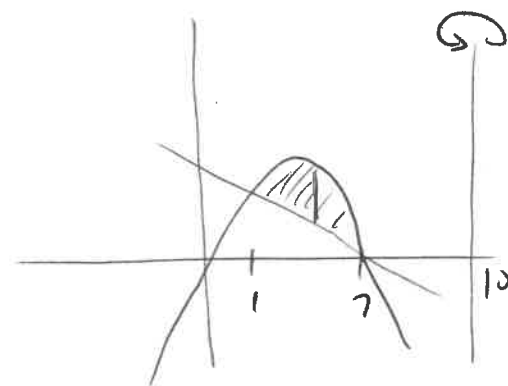
shell

$$r = 10 - x$$

$$h = (7x - x^2) - (7 - x)$$

$$= 7x - x^2 - 7 + x$$

$$= 8x - x^2 - 7$$

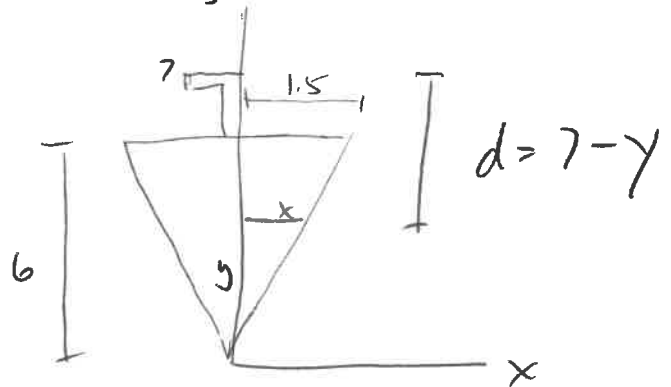
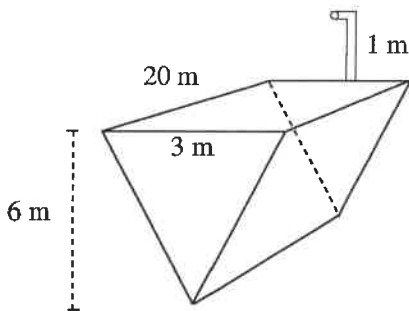


$$V = \int_1^7 2\pi (10 - x) (8x - x^2 - 7) dx$$

19. (10 points) A tank filled with water is in the shape of a trough with isosceles triangles at its ends. The trough is 20 meters long, has a height of 6 meters, and the width of the trough across the top is 3 meters. The trough has a spout with height 1 meter. The weight density of water is $\rho g = 9800 \text{ N/m}^3$.

Set up an integral that will compute the work required to pump all the water out of the spout. Do not evaluate!

Clearly indicate on the picture where you are placing the axis and which direction is positive.



Vol. of slice

$$\begin{aligned} V &= (2x)(l) \Delta y = 2x(20) \Delta y \\ &= 40x \Delta y \\ &= 40\left(\frac{1}{4}y\right) \Delta y \\ &= 10y \Delta y \end{aligned}$$

$$\frac{x}{y} = \frac{1.5}{6}$$

$$\frac{x}{y} = \frac{15}{60} = \frac{1}{4}$$

$$x = \frac{1}{4}y$$

$$F = \rho g V = \rho g 10y \Delta y$$

$$W = \int_0^6 F d = \int_0^6 \rho g 10y (7-y) dy$$

or

$$\int_0^6 9800 \cdot 10y (7-y) dy$$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-15		60
16		6
17		8
18		16
19		10
		100