MATH 152, Fall 2022

COMMON EXAM II - VERSION B

LAST NAME(print): ______________________ FIRST NAME(print): ______________________

INSTRUCTOR: ______________________

UIN: ______________________

SECTION NUMBER: ______________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to *fill in your name, UIN, section number and version letter of the exam* on the *ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

Some integrals that may or may not be useful.

$$\int \sec^3 x \, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

YMLNWK $\displaystyle\int \csc^3 x \, dx = \frac{-1}{2}\csc x \cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$

PART I: Multiple Choice. 3.5 points each

1. The sequence $a_n = \dfrac{(-1)^n n^2}{2n^2+5}$

   (a) Diverges

   (b) Converges to $\dfrac{1}{2}$

   (c) None of these.

   (d) Converges to 0

   (e) Converges to $\dfrac{-1}{2}$

2. Which of the following is an appropriate substitution to use when solving the integral $\int \sqrt{16x^2-9}\,dx$?

   (a) $x = \dfrac{3}{4}\tan\theta$

   (b) $x = \dfrac{3}{4}\sin\theta$

   (c) $x = \dfrac{4}{3}\sec\theta$

   (d) $x = \dfrac{4}{3}\sin\theta$

   (e) $x = \dfrac{3}{4}\sec\theta$

3. Which of the following is a proper Partial Fraction Decomposition for the rational function

$$\frac{5x+1}{(x+3)(x^2+4x+3)(x^2+4)}$$

   (a) $\dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+4x+3} + \dfrac{Dx+E}{x^2+4}$

   (b) $\dfrac{A}{x+3} + \dfrac{Bx+C}{(x+3)^2} + \dfrac{D}{x+1} + \dfrac{Ex+F}{x^2+4}$

   (c) $\dfrac{A}{x+3} + \dfrac{B}{(x+3)^2} + \dfrac{C}{x+1} + \dfrac{Dx+E}{x^2+4}$

   (d) $\dfrac{A}{x+3} + \dfrac{B}{(x+3)^2} + \dfrac{C}{x+1} + \dfrac{D}{x+2} + \dfrac{D}{x-2}$

   (e) None of these.

4. Assume that the sequence $\{a_n\}$ is decreasing and bounded below by 1, i.e. $a_n \geq 1$, for all positive n. Determine if the sequence is convergent or divergent.

$$a_1 = 4 \quad \text{and} \quad a_{n+1} = \frac{10}{7 - a_n}$$

(a) Convergent to 2
(b) Convergent to 1
(c) Divergent
(d) Convergent to $\frac{10}{7}$
(e) Convergent to 5

5. After an appropriate substitution, the integral $\int x^2\sqrt{9-x^2}\,dx$ is equivalent to which of the following?

(a) $81\int \sin^2\theta\cos^2\theta d\theta$
(b) $81\int \sec^3\theta\tan^2\theta\ d\theta$
(c) $27\int \sec^2\theta\tan\theta\ d\theta$
(d) $9\int \cos^2\theta\ d\theta$
(e) $27\int \sin^2\theta\cos\theta\ d\theta$

6. The series $\sum_{i=1}^{\infty}\left(e^{1/i} - e^{1/(i+1)}\right)$

(a) converges to 0
(b) None of these.
(c) converges to $e-1$
(d) diverges
(e) converges to e

7. Let $s = \sum_{n=1}^{\infty} \frac{1}{n^3}$ Using The Remainder Estimate for the Integral Test, determine the smallest value of n that ensures that $R_n = s - s_n \leq \frac{1}{44}$.

(a) n = 7
(b) n = 8
(c) n = 6
(d) n = 5
(e) n = 4

8. Compute $\int_0^4 \frac{x+2}{x^2+4}\, dx$.

(a) $\ln 20 - \ln 4$
(b) $\frac{1}{2}(\ln 20 - \ln 4) + \arctan(2)$
(c) $\ln 6 - \ln 2$
(d) $\frac{1}{2}(\ln 20 - \ln 4) + 2\arctan(4)$
(e) $\ln 20 - \ln 4 + 2\arctan(4)$

9. Let $\sum_{n=1}^{\infty} a_n$ be a series whose nth partial sum is $s_n = \frac{n}{n+2}$. Find a_4.

(a) $a_4 = \frac{2}{3}$
(b) None of these.
(c) $a_4 = \frac{1}{21}$
(d) $a_4 = \frac{1}{15}$
(e) $a_4 = 1$

10. Let $\sum_{n=1}^{\infty} a_n$ be a series whose nth partial sum is $s_n = \dfrac{7n^2+5}{5n^2+2}$. The series

(a) None of these.

(b) converges to $\dfrac{12}{7}$

(c) converges to 2.5

(d) diverges

(e) converges to $\dfrac{7}{5}$

11. Which sequence is both bounded and increasing?

(a) $a_n = \sin(2n\pi)$

(b) $a_n = 1 - \dfrac{2}{n}$

(c) $a_n = \ln n$

(d) $a_n = e^{-n}$

(e) None of these.

12. Compute $\displaystyle\int_{-1}^{\infty} \frac{1}{1+x^2}\,dx$.

(a) $\dfrac{\pi}{4}$

(b) None of these.

(c) ∞

(d) $\dfrac{3\pi}{4}$

(e) $\dfrac{\pi}{2}$

13. The sequence $a_n = \dfrac{n^2}{n+2} - \dfrac{n^2}{n+5}$

(a) Converges to 3

(b) Converges to 0

(c) None of these.

(d) Diverges

~~(e)~~ Converges to 7

14. Compute the sum of the series $\sum_{n=1}^{\infty} \frac{(-4)^{n+1}}{5^n}$.

(a) $\frac{16}{9}$
(b) $\frac{-20}{9}$
(c) None of these.
(d) $\frac{-16}{9}$
(e) This series diverges.

15. Which of the following statements is true regarding the improper integral $\int_1^{\infty} \frac{1}{e^x + \sqrt{x}}\, dx$?

(a) The integral converges because $\int_1^{\infty} \frac{1}{e^x + \sqrt{x}}\, dx < \int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$ and $\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$ converges.

(b) The integral diverges because $\int_1^{\infty} \frac{1}{e^x + \sqrt{x}}\, dx > \int_1^{\infty} \frac{1}{e^x}\, dx$ and $\int_1^{\infty} \frac{1}{e^x}\, dx$ diverges.

(c) The integral converges because $\int_1^{\infty} \frac{1}{e^x + \sqrt{x}}\, dx < \int_1^{\infty} \frac{1}{e^x}\, dx$ and $\int_1^{\infty} \frac{1}{e^x}\, dx$ converges.

(d) The integral diverges because $\int_1^{\infty} \frac{1}{e^x + \sqrt{x}}\, dx > \int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$ and $\int_1^{\infty} \frac{1}{\sqrt{x}}\, dx$ diverges.

(e) The integral converges to 0.

16. Which of the following series diverges by the Test for Divergence?

(I) $\sum_{n=1}^{\infty} \cos\left(\frac{\pi n}{2n+1}\right)$ (II) $\sum_{n=1}^{\infty} \frac{4}{4+e^{-3n}}$ (III) $\sum_{n=1}^{\infty} \frac{1}{\arctan n}$

(a) (III) only
(b) (II) and (III) only
(c) (I) and (II) only
(d) (II) only
(e) (I), (II), and (III)

17. Which of these substitutions would be used to evaluate $\int x^2\sqrt{x^2+4x+13}\,dx$?

(a) $x+2=3\sec\theta$
(b) $x^2+4x=\sqrt{13}\tan\theta$
(c) $x+4=\sqrt{13}\sec\theta$
(d) none of these.
(e) $x+2=3\tan\theta$

18. The improper integral $\int_1^e \frac{1}{x\ln x}\,dx$

(a) converges to $\frac{1}{e}-1$.
(b) diverges to ∞.
(c) converges to 1.
(d) converges to -1.
(e) diverges to $-\infty$.

PART II WORK OUT

Directions: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

19. (6 points) Find a general formula, a_n, for the sequence. Assume the pattern continues, and begins with $n=1$.

$$\left\{\frac{5}{8}, \frac{-9}{27}, \frac{13}{64}, \frac{-17}{125}, \frac{21}{216}, \dots\right\}$$

Sequence

5 9 13 17 21

4 4 4

$4n+1$

$$a_n = \frac{(-1)^{n+1}(4n+1)}{(n+1)^3}$$

20. (5 points) Determine whether the series converges or diverges. Fully support your conclusion.

$$\sum_{n=1}^{\infty} \frac{3^{2n}}{5^n} = \frac{3^2}{5^1} + \frac{3^4}{5^2} + \frac{3^6}{5^3} + \cdots$$

$a = \frac{9}{5}$ $\quad r = \frac{3^2}{5} = \frac{9}{5}$ $\quad$ Since $|r| = \frac{9}{5} > 1$

The geometric series will diverge.

21. (6 points) Determine whether the series converges or diverges. Fully support your conclusion.

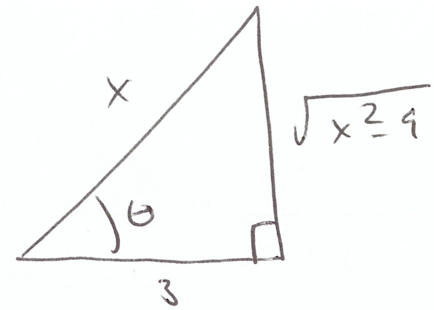
$$\sum_{n=1}^{\infty} ne^{-n^2}$$

See version A for this solution

22. (10 points) Compute $\int \frac{1}{x^4\sqrt{x^2-9}}\,dx$. In your final answer, any trig or inverse trig expressions that can be rewritten algebraically must be.

Let $x = 3\sec\theta \longrightarrow \sec\theta = \frac{x}{3}$

$dx = 3\sec\theta\tan\theta\,d\theta$



$$\int \frac{3\sec\theta\tan\theta}{(3\sec\theta)^4\sqrt{9\sec^2\theta-9}}\,d\theta = \int \frac{3\sec\theta\tan\theta}{3^4\sec^4\theta\sqrt{9\tan\theta}}\,d\theta$$

$$= \int \frac{3\sec\theta\tan\theta}{3^4\sec^4\theta\cdot 3\tan\theta}\,d\theta = \int \frac{1}{3^4}\cdot\frac{1}{\sec^3\theta}\,d\theta = \frac{1}{81}\int \cos^3\theta\,d\theta$$

$$= \frac{1}{81}\int \cos\theta\cdot\cos^2\theta\,d\theta = \frac{1}{81}\int \cos\theta\cdot(1-\sin^2\theta)\,d\theta$$

Let $u = \sin\theta$
$du = \cos\theta\,d\theta$

$$= \frac{1}{81}\int 1-u^2\,du = \frac{1}{81}\left[u - \frac{u^3}{3}\right] + C$$

$$= \frac{1}{81}\left[\sin\theta - \frac{\sin^3\theta}{3}\right] + C$$

$$= \frac{1}{81}\left[\frac{\sqrt{x^2-9}}{x} - \frac{1}{3}\left(\frac{\sqrt{x^2-9}}{x}\right)^3\right] + C$$

23. (10 points) Compute $\int \frac{x^2+7x+9}{(x+2)(x+1)^2}\,dx$

$$\frac{x^2+7x+9}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$x^2+7x+9 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$$
$$= A(x^2+2x+1) + B(x^2+3x+2) + C(x+2)$$

equating coeff

x^2] $1 = A + B$

x] $7 = 2A + 3B + C$

const] $9 = A + 2B + 2C$

shortcut: (evaluate #s)

Let $x = -2$ |

$4 - 14 + 9 = A(-1)^2$

$-1 = A$

Since $A + B = 1$

$-1 + B = 1$

$B = 2$

$7 = 2(-1) + 3(2) + C$

$7 = -2 + 6 + C$

$7 = 4 + C$

$C = 3$

$$\int \frac{-1}{x+2} + \frac{2}{x+1} + \frac{3}{(x+1)^2}\,dx$$

$$= -\ln|x+2| + 2\ln|x+1| + \frac{-3}{x+1} + C$$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-18		63
19		6
20		5
21		6
22		10
23		10
		100