MATH 152, Spring 2022

EXAM I - VERSION A

LAST NAME(print): solutions FIRST NAME(print): ____________

INSTRUCTOR: ____________

SECTION NUMBER: ____________

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to** ***fill in and correct bubble your name, UIN, section number and version letter of the exam on the ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________

PART I: Multiple Choice. 4 points each.

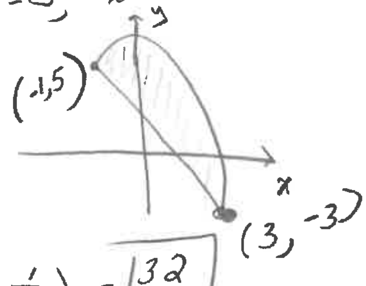
1. Find the area bounded by $y + x^2 = 6$ and $y + 2x - 3 = 0$.

   (a) $\frac{16}{3}$
   (b) $\frac{40}{3}$
   (c) $\frac{20}{3}$
   (d) $\frac{32}{3}$
   (e) None of the above

$y = 6 - x^2$ $\quad$ $6 - x^2 = 3 - 2x$

$y = 3 - 2x$ $\quad$ $0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$ $\quad$ $x = 3,\ x = -1$



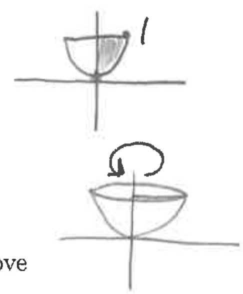
$A = \int_{-1}^{3} 6 - x^2 - (3 - 2x)\, dx$

$A = \int_{-1}^{3} (3 + 2x - x^2)\, dx$

$A = \left(3x + x^2 - \frac{x^3}{3}\right)\Big|_{-1}^{3} = 9 + 9 - 9 - \left(-3 + 1 + \frac{1}{3}\right) = \boxed{\frac{32}{3}}$

2. Consider the region R bounded by $y = 2x^2$ and $y = 1$, first quadrant only. Find the volume obtained by rotating R about the y-axis.

   (a) $\frac{\pi}{4}$
   (b) $\frac{\pi}{2}$
   (c) π
   (d) $\frac{4\pi}{5}$
   (e) None of the above



disk or shell

disk: $V = \int_0^1 \pi\left(\frac{y}{2}\right) dy$

$= \boxed{\frac{\pi}{4}}$

shell: $V = \int_0^{\sqrt{\frac{1}{2}}} 2\pi(x)(1 - 2x^2)\, dx$

$= 2\pi \int_0^{\sqrt{1/2}} (x - 2x^3)\, dx$

$= 2\pi\left(\frac{x^2}{2} - \frac{x^4}{2}\right)\Big|_0^{\sqrt{\frac{1}{2}}}$

$= 2\pi\left(\frac{1}{4} - \frac{1}{8}\right) = \boxed{\frac{\pi}{4}}$

3. Evaluate $\displaystyle\int_{\pi^2/16}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$

   (a) $\frac{\sqrt{2}}{2} + 1$
   (b) $2 + \sqrt{2}$
   (c) $\frac{\sqrt{2}}{2} - 1$
   (d) $-2 + \sqrt{2}$
   (e) $1 - \sqrt{2}$

u-sub

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\, dx$

$2\int_{\frac{\pi}{4}}^{\pi} \sin u\, du = -2\cos u \Big|_{\frac{\pi}{4}}^{\pi}$

$= -2\left(-1 - \frac{\sqrt{2}}{2}\right)$

$= \boxed{2 + \sqrt{2}}$

4. After an appropriate complete substitution, the integral $\int_{-1}^{4} \frac{x}{(x+7)^3}\,dx$ is equivalent to which of the following?

(a) $\int_6^{11} (7u^{-3} - u^{-2})\,du$

(b) $\int_{-1}^{4} (u^{-2} - 7u^{-3})\,du$

(c) $\int_6^{11} (u^{-2} - 7u^{-3})\,du$

(d) $\int_{-1}^{4} xu^{-3}\,du$

(e) $\int_6^{11} xu^{-3}\,du$

$u = x + 7$, so $x = u - 7$

$$\int_6^{11} \frac{u-7}{u^3}\,du = \int_6^{11} \left(u^{-2} - 7u^{-3}\right) du$$

5. If we revolve the region bounded by $y = 1 - x^2$ and $x - y = 1$ about the line $y = 3$, which of the following integrals gives the resulting volume?

(a) $\int_{-1}^{2} 2\pi(3-x)(x^2-x+2)\,dx$

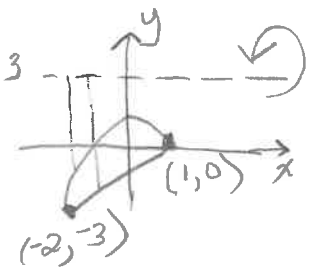
(b) $\int_{-2}^{1} \pi\left((2+x^2)^2 - (4-x)^2\right) dx$

(c) $\int_{-1}^{2} 2\pi(x-3)(x^2-x+2)\,dx$

(d) $\int_{-2}^{1} \pi\left((4-x)^2 - (2+x^2)^2\right) dx$

(e) $\int_{-1}^{2} \pi\left((2+x^2)^2 - (4-x)^2\right) dx$

$y = x - 1$ $\quad$ $1 - x^2 = x - 1$ $\quad$ washers!



$0 = x^2 + x - 2$

$= (x+2)(x-1)$

$R = 3 - (x-1) = 4 - x$

$r = 3 - (1 - x^2) = 2 + x^2$

$$V = \int_{-2}^{1} \pi\left[(4-x)^2 - (2+x^2)^2\right] dx$$

6. Find the area of the region bounded by $x = y^2$ and $x = y + 2$.

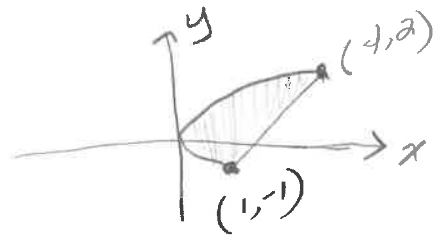
(a) $\frac{9}{2}$

(b) $\frac{3}{2}$

(c) $\frac{19}{6}$

(d) $\frac{16}{3}$

(e) None of the above



$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$$A = \int_{-1}^{2} y + 2 - y^2\,dy = \frac{y^2}{2} + 2y - \frac{y^3}{3}\Big|_{-1}^{2}$$

$= 2 + 4 - \frac{8}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= \boxed{\frac{9}{2}}$

we know $\sin^2 x = \frac{1}{2}(1-\cos(2x))$

7. Compute $\int 2\sin^2(2\theta)\,d\theta$ $= 2\int \sin^2(2\theta)\,d\theta$

$= \cancel{2}\int \frac{1}{\cancel{2}}(1-\cos(4\theta))\,d\theta$

$= \boxed{\theta - \frac{1}{4}\sin(4\theta) + C}$

(a) $\theta - \frac{1}{2}\sin(2\theta) + C$
(b) $\theta - \frac{1}{4}\sin(4\theta) + C$
(c) $\theta + \frac{1}{2}\sin(2\theta) + C$
(d) $\theta + \frac{1}{4}\sin(4\theta) + C$
(e) None of the above

8. Evaluate $\int_0^{\pi/4} \frac{\sec^2(\theta)}{2+\tan(\theta)}\,d\theta$

$u = 2 + \tan\theta$
$du = \sec^2\theta\,d\theta$

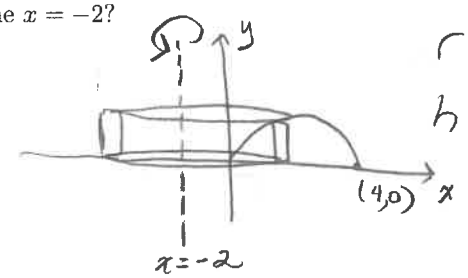
$\int_2^3 \frac{du}{u} = \ln u \Big|_2^3$

$= \ln 3 - \ln 2$

$= \ln\left(\frac{3}{2}\right)$

(a) $\ln\left(\frac{4}{3}\right)$
(b) $\ln\left(\frac{\pi}{4}\right)$
(c) $\ln\left(\frac{\pi}{8}\right)$
(d) $\ln\left(\frac{\pi}{12}\right)$
(e) $\ln\left(\frac{3}{2}\right)$

9. Consider the region R bounded by $y = 4x - x^2$ and $y = 0$. Which of the following integrals gives the volume of the solid obtained by revolving R about the line $x = -2$?



$r = x-(-2) = x+2$
$h = 4x - x^2$

(a) $\int_0^4 2\pi(2-x)(4x-x^2)\,dx$
(b) $\int_0^4 2\pi x(4x-x^2)\,dx$
(c) $\int_0^4 2\pi(x+2)(4x-x^2)\,dx$
(d) $\int_0^4 2\pi(x-2)(4x-x^2)\,dx$
(e) None of the above

$V = \int_0^4 2\pi(x+2)(4x-x^2)\,dx$

10. Evaluate $\int_1^2 \ln x \, dx$

(a) $2 \ln 2 - 1$
(b) $\frac{\ln(2)}{2} - 1$
(c) $\frac{\ln(2)}{2} - \frac{3}{2}$
(d) $2 \ln 2 - 3$
(e) $-\frac{1}{2}$

Parts: $u = \ln x$, $dv = dx$
$du = \frac{1}{x} dx$, $v = x$

$$\int_1^2 \ln x \, dx = x \ln x \Big|_1^2 - \int_1^2 dx$$
$$= (x \ln x - x)\Big|_1^2$$
$$= 2\ln 2 - 2 - (0 - 1)$$
$$= 2 \ln 2 - 1$$

11. A chain 30 meters long and weighing 24 newtons per meter hangs from the top of a 50 meter tall building. Calculate the work done in pulling the first 10 meters of this chain to the top of the building.

(a) 1200 Joules
(b) 3000 Joules
(c) 9000 Joules
(d) 6000 Joules
(e) None of the above

$$W = \int_0^{10} (30)(24) - 24x \, dx$$
$$= 720x - 12x^2 \Big|_0^{10}$$
$$= \boxed{6000 \text{ J}}$$

12. A force of 40 N is required to hold a spring that has been stretched from its natural length of 1 m to a length of 3 m. Find the work required to stretch this spring from a length of 4 meters to a length of 5 meters.

(a) 90 Joules
(b) 70 Joules
(c) 80 Joules
(d) 100 Joules
(e) None of the above

Given $f(2) = 40$, where $f(x) = kx$
$2k = 40$
$k = 20$

$$W = \int_3^4 20x \, dx$$
$$= 10x^2 \Big|_3^4$$
$$= \boxed{70 \text{ J}}$$

13. Compute $\int_0^{\pi/3} \tan^3(\theta)\sec(\theta)\, d\theta$ $= \int_0^{\frac{\pi}{3}} \tan^2\theta \sec\theta \tan\theta \, d\theta$

(a) $\frac{4}{3}$

(b) $\frac{16 - 9\sqrt{3}}{24}$

(c) $\frac{2}{3}$

(d) $\frac{-3\sqrt{3}}{8}$

(e) None of the above

$= \int_0^{\frac{\pi}{3}} (\sec^2\theta - 1)\sec\theta \tan\theta \, d\theta$

$u = \sec\theta$

$du = \sec\theta \tan\theta \, d\theta$

$\int_1^2 (u^2 - 1)\, du = \frac{u^3}{3} - u \Big|_1^2$

$= \frac{8}{3} - 2 - \frac{1}{3} + 1$

$= \boxed{\frac{4}{3}}$

14. Compute $\int_0^1 xe^{-x}\, dx$

(a) $1 + 2e^{-1}$

(b) $\frac{1}{2} - \frac{1}{2}e^{-1}$

(c) $2e^{-1} - 1$

(d) $-\frac{1}{2} + \frac{1}{2}e^{-1}$

(e) $1 - 2e^{-1}$

parts:

$u = x \qquad dv = e^{-x}dx$

$du = dx \qquad v = -e^{-x}$

u	dv
x	e^{-x}
1	$-e^{-x}$
0	e^{-x}

$(-xe^{-x} - e^{-x}) \Big|_0^1$

$-e^{-1} - e^{-1} + 1 = \boxed{-2e^{-1} + 1}$

15. Find the area bounded by $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, and $x = 3$.

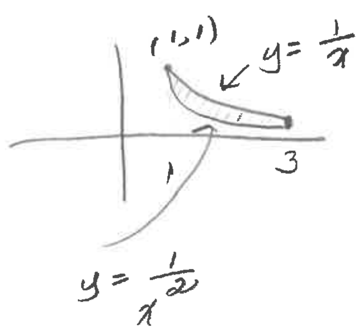
(a) $\frac{4}{9}$

(b) $\ln 3 + \frac{4}{3}$

(c) $\ln 3 + \frac{2}{3}$

(d) $\frac{2}{9}$

(e) $\ln 3 - \frac{2}{3}$



$A = \int_1^3 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

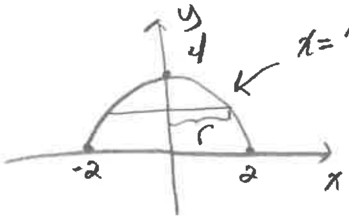
$= \ln|x| + \frac{1}{x} \Big|_1^3$

$= \ln 3 + \frac{1}{3} - 1$

$= \boxed{\ln 3 - \frac{2}{3}}$

PART II: Free Response: Show all work and box your final answer!

16. (8 pts) Consider the solid S whose base is the region bounded by $y = 4 - x^2$ and $y = 0$. Cross sections perpendicular to the y - axis are semicircles. Find the volume of S.



$$V = \int_0^4 A(y)\,dy, \quad \text{where } A(y) = \tfrac{1}{2}\pi r^2$$
$$\text{and } r = \sqrt{4-y}$$

$$A(y) = \tfrac{1}{2}\pi(4-y)\,dy$$

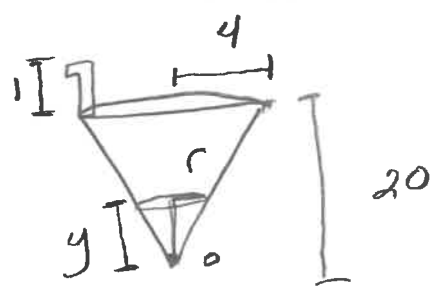
$$V = \int_0^4 \tfrac{\pi}{2}(4-y)\,dy$$

$$V = \tfrac{\pi}{2}\left(4y - \tfrac{y^2}{2}\right)\Big|_0^4$$

$$= \tfrac{\pi}{2}(16-8)$$

$$\boxed{V = 4\pi}$$

17. (9 pts) A tank is in the shape of an inverted cone with radius $r = 4$ feet and height $h = 20$ feet. Assuming it is full of water, set up but **do not evaluate** an integral that gives the work it takes to pump the water through a 1 foot tall spout located at the top of the tank. Use $\rho g = 62.5$ pounds per cubic foot. **Clearly indicate where you are placing the axis and which direction is positive.**



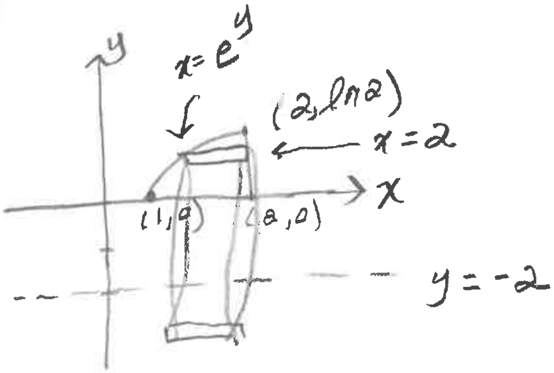
$$V_i = \pi r^2 dy, \quad \text{where } \frac{r}{y} = \frac{4}{20}$$
$$r = \tfrac{1}{5}y$$

$$V_i = \pi\left(\tfrac{1}{5}y\right)^2 dy$$

$$W = \int_0^{20} \tfrac{\pi}{25}\rho g y^2 (21-y)\,dy$$

18. Consider the region R bounded by $y = \ln x$, $y = 0$, and $x = 2$. If this region is revolved about the line $y = -2$:

(a) (7 pts) Set up but **do not evaluate** the integral that gives the volume using the method of shells.

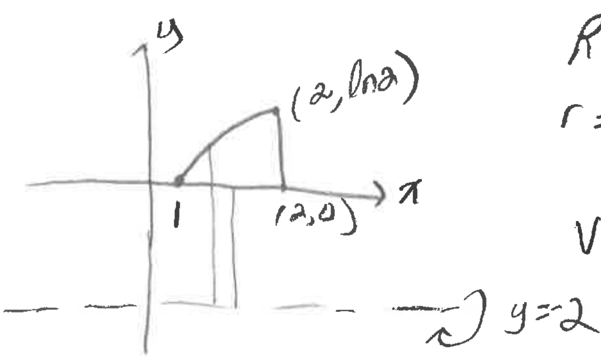


$$V = \int_0^{\ln 2} 2\pi (y+2)(2-e^y)\,dy$$

$h = 2 - e^y$

$r = y + 2$

(b) (7 pts) Set up but **do not evaluate** the integral that gives the volume using the method of washers.



$R = \ln x + 2$

$r = 2$

$$V = \int_1^2 \pi\left((\ln x + 2)^2 - 4\right) dy$$

19. (9 pts) Find $\int e^{2x} \cos x \, dx$ parts, where ① $u = e^{2x}$ or ② $u = \cos x$

① $u = e^{2x}$ $dv = \cos x \, dx$

$du = 2e^{2x} dx$ $v = \sin x$

$\int e^{2x} \cos x \, dx = e^{2x} \sin x - \int 2e^{2x} \sin x \, dx$ parts agan

$u = 2e^{2x}$, $dv = \sin x \, dx$

$du = 4e^{2x} dx$, $v = -\cos x$

$= e^{2x} \sin x - \left(-2e^{2x} \cos x + \int 4e^{2x} \cos x \, dx \right)$

$= e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x \, dx$

$5 \int e^{2x} \cos x \, dx = e^{2x} \sin x + 2e^{2x} \cos x + C$, so

$$\boxed{\int e^{2x} \cos x \, dx = \frac{1}{5}\left(e^{2x} \sin x + 2e^{2x} \cos x\right) + C}$$

② $u = \cos x$, $dv = e^{2x} dx \longrightarrow du = -\sin x \, dx$, $v = \frac{1}{2} e^{2x}$

$\int e^{2x} \cos x \, dx = \frac{1}{2} e^{2x} \cos x + \int \frac{1}{2} e^{2x} \sin x \, dx$

parts again, $u = \sin x$, $dv = \frac{1}{2} e^{2x} dx$

$du = \cos x \, dx$, $v = \frac{1}{4} e^{2x}$

$= \frac{1}{2} e^{2x} \cos x + \frac{1}{4} e^{2x} \sin x - \int \frac{1}{4} e^{2x} \cos x \, dx$

$\frac{5}{4} \int e^{2x} \cos x \, dx = \frac{1}{2} e^{2x} \cos x + \frac{1}{4} e^{2x} \sin x$

$\int e^{2x} \cos x \, dx = \frac{4}{5}\left(\frac{1}{2} e^{2x} \cos x + \frac{1}{4} e^{2x} \sin x\right) + C$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-15		60
16		8
17		9
18		14
19		9
		100