MATH 152, Spring 2022

EXAM II - VERSION B

LAST NAME(print): Key ______ FIRST NAME(print): ______

UIN: ______

INSTRUCTOR: ______

SECTION NUMBER: ______

DIRECTIONS:

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the **correct** choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the quality and **correct**ness of the work leading up to it.
5. **Be sure to** ***fill in your name, UIN, section number and version letter of the exam on the ScanTron form.***

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______

PART I: Multiple Choice. 4 points each.

1. Which of the following series diverges by the Test for Divergence?

(a) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

(b) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

(d) $\sum_{n=1}^{\infty} e^{-n}$

(e) Test for Divergence fails for all of the above series. **correct**

$\lim_{n\to\infty} a_n = 0$ for all a_n, so test for divergence fails for all.

2. Which of the following is a proper Partial Fraction Decomposition for $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)}$?

(a) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{x+4} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$ **correct**

(b) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{Ax+B}{x^2-16} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

(c) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{Ax+B}{x^2-16} + \frac{Cx+D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

(d) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{x+4} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{E}{x^2+1}$

(e) $\frac{x+1}{(x^2-16)(x-3)^2(x^2+1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$

$$\frac{x+1}{(x+4)(x-4)(x-3)(x-3)^2(x^2+1)} = \frac{A}{x+4} + \frac{B}{x-4} + \frac{C}{x-3} + \frac{D}{(x-3)^2} + \frac{Ex+F}{x^2+1}$$

3. Which of the following statements is/are true for the sequences shown below?

(I) $a_n = \cos\left(\frac{1}{n}\right)$ (II) $a_n = \frac{\cos(n)}{n}$

(a) (I) converges to 0, and (II) converges to zero.

(b) (I) converges to 1, and (II) converges to zero. **correct**

(c) (I) diverges, and (II) converges to zero.

(d) (I) converges to 1, and (II) diverges

(e) Both diverge

$\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos(0) = 1$

$\lim_{n\to\infty} \frac{\cos(n)}{n^2} = \frac{\text{number between } \pm 1}{\infty}$

$= 0$

4. Which of the following integrals are improper?

(I) $\int_0^1 \frac{1}{3x-1}\,dx$ (II) $\int_1^3 \ln(x-1)\,dx$ (III) $\int_{-\infty}^1 \frac{1}{x^4}\,dx$

(a) (III) only
(b) (I) and (III) only
(c) (II) and (III) only
(d) (I) and (II) only
(e) All of them are improper. **correct**

I. $\frac{1}{3x-1}$ has a vertical asymptote at $x=\frac{1}{3}$, which is on $[0,1]$

II. $\ln(x-1)$ has a vertical asymptote at $x=1$, which is on $[1,3]$

III. $\frac{1}{x^4}$ has a limit of integration of $-\infty$

5. Consider the recursive sequence $a_1 = 2$, $a_{n+1} = 5 - \frac{4}{a_n}$. Given the sequence is increasing and bounded, find the limit.

ensures limit exists

(a) 1
(b) 4 **correct**
(c) 2
(d) $\frac{5}{2}$
(e) 5

$a_1 = 2$

Let $L = \lim_{n\to\infty} a_n$. then it follows that

$L = \lim_{n\to\infty} a_{n+1}$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\left(5 - \frac{4}{a_n}\right)$$

$$L = 5 - \frac{4}{L}$$

$L^2 = 5L - 4$

$L^2 - 5L + 4 = 0$

$(L-4)(L-1) = 0$

$L = 4$

6. Which of the following statements is true regarding the improper integral $\int_1^\infty \frac{\cos^2 x + 5}{x^4}\,dx$?

(a) It converges since $\int_1^\infty \frac{\cos^2 x+5}{x^4}\,dx \le \int_1^\infty \frac{5}{x^4}\,dx$, which converges.
(b) It converges since $\int_1^\infty \frac{\cos^2 x+5}{x^4}\,dx \le \int_1^\infty \frac{1}{x^4}\,dx$, which converges.
(c) It diverges by oscillation.
(d) It converges to zero.
(e) It converges since $\int_1^\infty \frac{\cos^2 x+5}{x^4}\,dx \le \int_1^\infty \frac{6}{x^4}\,dx$, which converges. **correct**

$$\int_1^\infty \frac{\cos^2 x + 5}{x^4} \le \int_1^\infty \frac{6}{x^4},$$ which converges since $p = 4 > 1$.

CT says therefore both improper integrals converge

7. Find the limit of the sequence $a_n = \arcsin\left(\frac{n+1}{1-2n}\right)$

(a) $-\frac{\pi}{3}$

(b) $\frac{4\pi}{3}$

(c) $-\frac{\pi}{6}$ **correct**

(d) $\frac{7\pi}{6}$

(e) $\frac{\pi}{2}$

$$\lim_{n\to\infty} a_n = \arcsin\left(\lim_{n\to\infty} \frac{n+1}{1-2n}\right)$$
$$= \arcsin\left(-\frac{1}{2}\right)$$
$$= \frac{-\pi}{6}$$

8. After an appropriate trigonometric substitution, $\int_{2\sqrt{2}}^{4} \frac{\sqrt{x^2-4}}{x}\,dx$ is equivalent to

(a) $\int_{\pi/4}^{\pi/6} \sin(\theta)\,d\theta$

(b) $\int_{\pi/4}^{\pi/3} \sin(\theta)\,d\theta$

(c) $2\int_{\pi/4}^{\pi/6} \tan^2\theta\,d\theta$

(d) $2\int_{\pi/4}^{\pi/3} \tan^2\theta\,d\theta$ **correct**

(e) None of the above

$x = 4,\ \theta = \frac{\pi}{3}$

$x = 2\sec\theta <$ $x = 2\sqrt{2},\ \theta = \frac{\pi}{4}$

$dx = 2\sec\theta\tan\theta\,d\theta$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sqrt{4\sec^2\theta - 4}}{\cancel{2\sec\theta}}\,\cancel{2\sec\theta}\tan\theta\,d\theta$$

$$2\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2\theta\,d\theta$$

9. $\int \frac{x^3+x}{x-1}\,dx =$

(a) $\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln|x-1|+C$ **correct**

(b) $\frac{x^3}{3}-\frac{x^2}{2}+2x+2\ln|x-1|+C$

(c) $\frac{x^3}{3}+\frac{x^2}{2}+2x-2\ln|x-1|+C$

(d) $\frac{x^3}{3}+\frac{x^2}{2}-2x+2\ln|x-1|+C$

(e) $\frac{x^3}{3}-\frac{x^2}{2}+2x-2\ln|x-1|+C$

$$\begin{array}{r|l} & x^2 + x + 2 \\ \hline x-1 & x^3 + x \\ & -(x^3 - x^2) \\ & x^2 + x \\ & -(x^2 - x) \\ & 2x \\ & -(2x - 2) \\ & 2 \end{array}$$

$$\int x^2 + x + 2 + \frac{2}{x-1}\,dx \rightarrow \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln|x-1| + C$$

10. Suppose $s_n = \dfrac{3n+4}{2n+2}$ is the sequence of partial sums for the series $\sum\limits_{n=1}^{\infty} a_n$. Find $\sum\limits_{n=1}^{\infty} a_n$ and a_4.

(a) $\sum\limits_{n=1}^{\infty} a_n = \dfrac{3}{2}$ and $a_4 = -\dfrac{1}{60}$

(b) $\sum\limits_{n=1}^{\infty} a_n = \dfrac{3}{2}$ and $a_4 = -\dfrac{1}{40}$ **correct**

(c) $\sum\limits_{n=1}^{\infty} a_n = 2$ and $a_4 = -\dfrac{1}{40}$

(d) $\sum\limits_{n=1}^{\infty} a_n = \dfrac{3}{2}$ and $a_4 = -\dfrac{129}{40}$

(e) $\sum\limits_{n=1}^{\infty} a_n = 2$ and $a_4 = -\dfrac{1}{60}$

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{3n+4}{2n+2} = \frac{3}{2}$$

$$a_4 = s_4 - s_3 = \frac{16}{10} - \frac{13}{8} = -\frac{1}{40}$$

11. After an appropriate trigonometric substitution, $\displaystyle\int \frac{dx}{\sqrt{x^2+8x+41}}$ is equivalent to which of the following?

(a) $\dfrac{1}{5}\displaystyle\int \cos(\theta)\,d\theta$

(b) $\dfrac{1}{5}\displaystyle\int \sin(\theta)\,d\theta$

(c) $\displaystyle\int \sec^2(\theta)\,d\theta$

(d) $\displaystyle\int \tan(\theta)\,d\theta$

(e) $\displaystyle\int \sec(\theta)\,d\theta$ **correct**

complete square: $\underbrace{x^2+8x+16}_{(x+4)^2} \underbrace{+41-16}_{25}$

$$\int \frac{dx}{\sqrt{(x+4)^2+25}}$$

$x+4 = 5\tan\theta$

$dx = 5\sec^2\theta\, d\theta$

$$\int \frac{5\sec^2\theta\, d\theta}{\sqrt{25\tan^2\theta+25}} = \int \frac{5\sec^2\theta\, d\theta}{5\sec\theta} = \int \sec\theta\, d\theta$$

12. Evaluate $\displaystyle\int_1^{\infty} \frac{e^{2/x}}{x^2}\,dx$.

(a) $-2\left(1-e^2\right)$

(b) $2\left(1-e^2\right)$

(c) $-\dfrac{1}{2}\left(1-e^2\right)$ **correct**

(d) $\dfrac{1}{2}\left(1-e^2\right)$

(e) $\dfrac{1}{2}e^2$

$$\lim_{t\to\infty} \int_1^t \frac{e^{\frac{2}{x}}}{x^2}\,dx$$

$u = \frac{2}{x}$ $\quad x=t,\ u=\frac{2}{t}$ $\quad x=1,\ u=2$

$du = \frac{-2}{x^2}\,dx$

$$\lim_{t\to\infty} \int_2^{\frac{2}{t}} -\frac{1}{2} e^u\,du = \lim_{t\to-\infty} -\frac{1}{2}\left(e^{\frac{2}{t}} - e^2\right)$$

$$= -\frac{1}{2}\left(1-e^2\right)$$

13. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{4^n} =$

$$\sum_{n=1}^{\infty} -3\left(-\frac{3}{4}\right)^n = \sum_{n=1}^{\infty} \frac{9}{4}\left(-\frac{3}{4}\right)^{n-1}$$

$$= \frac{9/4}{1+3/4}$$

$$= \frac{9}{7}$$

(a) 9

(b) $\frac{2}{3}$

(c) $\frac{9}{7}$ **correct**

(d) $-\frac{12}{7}$

(e) -12

14. The sequence $a_n = \frac{1}{3}\ln(4+2n) - \frac{1}{3}\ln(4n+1)$ $= \frac{1}{3}\ln\left(\frac{4+2n}{4n+1}\right)$

(a) Converges to $\frac{1}{3}\ln(2)$

(b) Diverges

(c) Converges to 0

(d) Converges to $\frac{1}{3}\ln(4)$

(e) Converges to $\frac{1}{3}\ln\left(\frac{1}{2}\right)$ **correct**

$$\lim_{n\to\infty} \frac{1}{3}\ln\left(\frac{4+2n}{4n+1}\right)$$

$$= \frac{1}{3}\ln\left(\lim_{n\to\infty} \frac{4+2n}{4n+1}\right) = \frac{1}{3}\ln\left(\frac{1}{2}\right)$$

True or False. On your scantron, bubble 'a' if true and bubble 'b' if false. One point each.

15. Given $f(x)$ and $g(x)$ are continuous, positive functions on the interval $[a,\infty)$ and $f(x) \geq g(x)$ on the interval $[a,\infty)$. If $\int_a^{\infty} g(x)\,dx$ diverges, so does $\int_a^{\infty} f(x)\,dx$. **True**

16. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges. **False**

17. If $\lim_{n\to\infty} s_n = 1$, then $\sum_{n=1}^{\infty} a_n$ diverges. **False**

18. The geometric series $\sum_{n=2}^{\infty} ar^{n-1}$ converges if $|r| < 1$. **True**

PART II: Free Response: Show all work and box your final answer!

19. (10 pts) $\int \frac{2x^2+2x+1}{x^2(x^2+1)}\,dx$

$$\frac{2x^2+2x+1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$2x^2+2x+1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$$

$x=0$: $\boxed{1 = B}$

$$2x^2+2x+1 = Ax^3 + Ax + x^2 + 1 + Cx^3 + Dx^2$$

$$= (A+C)x^3 + (1+D)x^2 + Ax + 1$$

$A + C = 0$ $\quad \boxed{2 = A}$

$2 = 1 + D$ $\quad \boxed{C = -2}$

$\boxed{D = 1}$

$$\int\left(\frac{2}{x} + \frac{1}{x^2} + \frac{-2x+1}{x^2+1}\right)dx$$

$$= \int\left(\frac{2}{x} + \frac{1}{x^2} - \frac{2x}{x^2+1} + \frac{1}{x^2+1}\right)dx$$

$$= 2\ln|x| - \frac{1}{x} - \ln(x^2+1) + \arctan x + C$$

20. (10 pts) Find $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$\int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} \, 3\cos\theta\, d\theta$$

$$\int \frac{27\sin^2\theta\cos\theta}{\sqrt{9\cos^2\theta}}\, d\theta$$

$$\int \frac{27\sin^2\theta\cos\theta}{3\cos\theta}\, d\theta$$

$$9\int \sin^2\theta\, d\theta$$

$$\frac{9}{2}\int (1-\cos 2\theta)\, d\theta$$

$$\frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

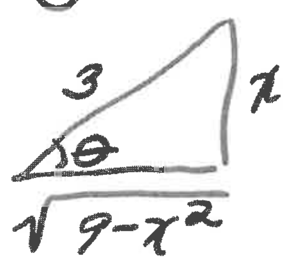
$$\frac{9}{2}\left(\theta - \frac{1}{2}(2\sin\theta\cos\theta)\right) + C$$

$$\frac{9}{2}\left(\theta - \sin\theta\cos\theta\right) + C$$

$$\frac{9}{2}\left(\arcsin\frac{x}{3} - \frac{x}{3}\,\frac{\sqrt{9-x^2}}{3}\right) + C$$

$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$



21. Consider the series $\sum_{n=1}^{\infty} \frac{1}{(n+5)[\ln(n+5)]^2}$

Let $f(x) = \frac{1}{(x+5)[\ln(x+5)]^2}$ a continuous, positive decreasing function.

(a) (6 pts) Prove the series converges.

$$\int_1^{\infty} \frac{dx}{(x+5)[\ln(x+5)]^2}$$

$u = \ln(x+5)$

$du = \frac{1}{x+5}\,dx$

$$\lim_{t\to\infty} \int_{\ln 6}^{\ln(t+5)} \frac{du}{u^2}\,du$$

$$\lim_{t\to\infty} -\frac{1}{u}\Big|_{\ln 6}^{\ln(t+5)} = \lim_{t\to\infty}\left(\cancel{-\frac{1}{\ln(t+5)}}^{\,0} + \frac{1}{\ln 6}\right)$$

$$= \frac{1}{\ln 6}$$

integral converges, so does series by the integral test.

(b) (4 pts) Using the Remainder Estimate for the Integral Test, find an upper bound on the remainder, R_6, if we used s_6, the 6^{th} partial sum, to approximate the sum of the series.

$$R_6 \le \int_6^{\infty} \frac{dx}{(x+5)[\ln(x+5)]^2}$$

import integral from part (a)

$$= -\frac{1}{\ln(x+5)}\Big|_6^{\infty}$$

$$= \frac{1}{\ln(11)}, \text{ so } R_6 \le \frac{1}{\ln(11)}$$

22. Consider the series $\sum_{n=1}^{\infty} \frac{6}{n(n+2)}$

(a) (6 pts) Find a formula for s_n, the n^{th} partial sum.

PFD: $\frac{6}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$

$n=0$

$6 = A(2)$ $\boxed{A=3}$

$6 = A(n+2) + Bn$

$n=-2$

$6 = B(-2)$ $\boxed{B=-3}$

Find Sn for $\sum_{n=1}^{\infty} \left(\frac{3}{n} - \frac{3}{n+2} \right)$

$S_n = 3 - \frac{3}{3} + \frac{3}{2} - \frac{3}{5} + \frac{3}{3} - \frac{3}{6} + \cdots + \frac{3}{n-1} - \frac{3}{n+1} + \frac{3}{n} - \frac{3}{n+2}$

$\boxed{S_n = 3 + \frac{3}{2} - \frac{3}{n+1} - \frac{3}{n+2}}$

(b) (4 pts) Find the sum of the series.

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(3 + \frac{3}{2} - \frac{3}{n+1} - \frac{3}{n+2} \right)$

$= 3 + \frac{3}{2}$ or $\frac{9}{2}$

DO NOT WRITE IN THIS TABLE.

Question	Points Awarded	Points
1-18		60
19		10
20		10
21		10
22		10
		100