# Spring 1999
# Math 152
# Common Exam 3
# Test Form A

**PRINT: Last Name:** ______________________________ **First Name:** ______________________

**Signature:** __________________________________________ **ID:** _________________________

**Instructor's Name:** __________________________________________ **Section #** _____________

## INSTRUCTIONS

1. In **Part 1** (Problems 1–8), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will be collected all at once after 1 hour; they will NOT be returned.
2. In **Part 2** (Problems 9–13), write all solutions in the space provided. You may use the back of any page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWERS.

## PART 1: MULTIPLE-CHOICE PROBLEMS

*Each problem is worth 5 points; NO partial credit will be given. Calculators may NOT be used during the first hour. ScanTrons will be collected after 1 hour.*

1. Find the *center* of the sphere whose equation is $x^2 + 2x + y^2 - y + z^2 = 0$.

   (a) $\left(-1, -\frac{1}{2}, 0\right)$ (b) $\left(1, -\frac{1}{2}, 0\right)$ (c) $(2, -1, 0)$ (d) $\left(-1, \frac{1}{2}, 0\right)$ (e) $(-2, 1, 0)$

2. Find the *length* of the vector $\mathbf{2a} - \mathbf{3b}$, where $\mathbf{a} = \langle 1, 1, 1 \rangle$ and $\mathbf{b} = \langle -1, 2, 0 \rangle$.

   (a) $3\sqrt{5}$ (b) $45$ (c) $\sqrt{13}$ (d) $1$ (e) $\frac{1}{\sqrt{15}}$

3. Find the *point of intersection* of the line $x = 1 + t,\ y = 1 - t,\ z = 1 + 2t$ and the plane $x + 2y + 3z = 11$.

   (a) $(0, 2, -1)$ (b) $(1, -1, 2)$ (c) $(1, 2, 3)$ (d) $(1, 2, 2)$ (e) $(2, 0, 3)$

4. Find *an equation* of the plane through the three points $A(-2,-8,0)$, $B(8,6,2)$, and $C(-5,4,-8)$.

(a) $Ax+By+Cz=0$ (b) $68x-37y-81z=160$ (c) $x+2y-6z=0$

(d) $-68x-37y+81z=-160$ (e) $(x-A)^2+(y-B)^2+(z-C)^2=(A+B+C)/2$

5. Find the *volume* of the parallelepiped (sheared box) determined by the three vectors

$$\mathbf{a}=\mathbf{i}+\mathbf{2j}, \qquad \mathbf{b}=\mathbf{3j}+\mathbf{4k}, \qquad \mathbf{c}=\mathbf{5i}+\mathbf{6k}$$

(a) 60 (b) 22 (c) 58 (d) 64 (e) 72

6. Find a *power series* representation for $f(x)=\dfrac{x}{x+5}$ and determine its *radius of convergence $R$*.

(a) $R=5,\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n!}\,x^{5n}$ (b) $R=1,\ \displaystyle\sum_{n=0}^{\infty}\frac{1}{5}\,x^{n+1}$ (c) $R=5,\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{5^n}\,x^{n}$

(d) $R=1,\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n!}\,x^{5n}$ (e) $R=1,\ \displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{5^n}\,x^{n}$

7. An equation of the *tangent line* to the curve $\mathbf{r}(t) = \langle \sin\left(\frac{\pi t}{2}\right),\ \ln(2t-1),\ 3t^2-1 \rangle$ at the point $(1, 0, 2)$ is:

(a) $\mathbf{L}(s) = \langle 1 + \frac{\pi}{2}s,\ 2s,\ 2-s \rangle$ (b) $\mathbf{L}(u) = \langle u,\ 0,\ 2u \rangle$ (c) $\mathbf{L}(s) = \langle 1 + \frac{\pi}{2}s,\ -2s,\ 2 \rangle$

(d) $\mathbf{L}(u) = \left\langle \cos\frac{\pi u}{2},\ \frac{1}{2u-1},\ 6u \right\rangle$ (e) $\mathbf{L}(s) = \langle 1,\ 2s,\ 2+6s \rangle$

8. Find the *vector projection* of $\mathbf{b}$ onto $\mathbf{a}$. Here $\mathbf{a} = -4\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(a) $\left\langle -\frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$ (b) $\left\langle -\frac{14}{9}, -\frac{7}{9}, \frac{14}{9} \right\rangle$ (c) $\langle 13, -14, 6 \rangle$ (d) $\left\langle \frac{14}{25}, \frac{7}{10}, \frac{21}{50} \right\rangle$ (e) $\langle 7, 2, -14 \rangle$

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## PART 2: WORK-OUT PROBLEMS

*Each problem is worth 12 points; partial credit is possible. Calculators are permitted ONLY during the second hour of the exam, AFTER the ScanTrons are collected.* **SHOW ALL STEPS IN YOUR WORK.** *If you use a calculator, explain how. In this regard, exact and/or floating point decimal answers are acceptable.*

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9. Find the *angle $\theta$ between the tangent vectors* to the space curves

$$\mathbf{r_1}(s) = \langle s,\ 2-s^2,\ s^4+4 \rangle, \quad s \geq 0; \qquad \mathbf{r_2}(t) = \langle t, t^2-2, 4+t^4 \rangle, \quad t \geq 0$$

at their unique point of intersection.

10. The nonparallel planes $P_1 : 2x - y + z = 1$ and $P_2 : x + 2y - 3z = -4$ intersect in a line.

(a) Where does this line intersect the $xy$-plane?

(b) Find a set of parametric equations for this line.

11. Find the *Maclaurin series* for $f(x) = \ln(1 - x)$ and determine its interval of convergence.

12. Let $f(x) = \sqrt{x}$.

(a) Find the 3rd degree Taylor polynomial $T_3(x)$ of $f(x)$ about $x = 4$.

(b) Use Taylor's Inequality to give an ***estimate*** for the maximum error in this approximation over the interval $3 < x < 5$.

13. In Einstein's special theory of relativity, the relativistic generalization of the kinetic energy of an object is given by

$$K = mc^2 \left( \left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1 \right) \qquad (\star)$$

Here $m$ is the object's mass, $c$ is the speed of light, and $v$ is the speed of the object. Show that for everyday speeds (i.e., whenever $v$ is VERY MUCH LESS than $c$), the expression $(\star)$ reduces to the classical kinetic energy of Newtonian theory, $K = \frac{1}{2}mv^2$. Proceed as follows.

(a) Compute the first 3 terms of the Maclaurin series for $(1 + x)^{-1/2}$.

(b) Substitute $x = -\dfrac{v^2}{c^2}$ into (a); this gives an expansion for $\left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$.

(c) Now plug (b) into $(\star)$ and expand. Since $v$ is very much less than $c$, all terms in the resulting series are negligible except the first. This leaves $K = \frac{1}{2}mv^2$.