MATH 152, Exam I, Version A
Fall, 1994

NAME

ID #

INSTRUCTOR'S NAME

SECTION #

INSTRUCTIONS

1. In Part I (problems 1-10), mark the correct choice on your SCANTRON sheet using a #2 pencil. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned.

2. In Part II (problems 11-20), write all solutions in the space provided. Use the back of each page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWER.

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Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS
The SCANTRON forms will be collected at the end of 1 hour.

1. Evaluate $\displaystyle\int_0^{\ln(2)} e^{2x}\,dx.$

(a) 1 (b) e^2 (c) 2 (D) $\dfrac{3}{2}$ (e) e^{-1}

2. Evaluate $\displaystyle\int_0^{\ln(5)} \frac{e^x}{1+e^x}\,dx$

(a) $4-\ln(3)$ (B) $\ln(3)$ (c) $\ln(1+\ln 5)$ (d) $\ln(5)$ (e) $\ln(5)-\ln(1+\ln 5)$

3. Evaluate $\displaystyle\int_0^{\pi/4} \sin^2(2\theta)\cos^3(2\theta)\,d\theta.$

(A) $\dfrac{1}{15}$ (b) $\dfrac{4}{15}$ (c) -1 (d) $\dfrac{\pi^3}{96}-\dfrac{\pi^5}{2560}$ (e) $\dfrac{\pi^3}{384}-\dfrac{\pi^5}{10240}$

4. Given that a and b are positive real numbers, which of the following is FALSE?

(a) $\ln(a\ b)=\ln(a)+\ln(b)$ (b) $a^{\ln(b)}=b^{\ln(a)}$ (c) $\ln(\sqrt{a})=\dfrac{1}{2}\ln(a)$

(D) $\dfrac{\ln(a)}{\ln(b)}=\ln\left(\dfrac{a}{b}\right)$ (e) $\ln(a)-\ln(b)=\ln\left(\dfrac{a}{b}\right)$

5. Evaluate $\displaystyle\lim_{t\to 0}\frac{(1-\cos t)\cdot\sin t}{t^3}.$

(a) 0 (B) $\dfrac{1}{2}$ (c) -1 (d) $-\dfrac{1}{2}$ (e) π

6. Evaluate $\displaystyle\int_0^1 \mathrm{e}^{-2x} \sinh x\, dx.$

(a) $\dfrac{1}{3\mathrm{e}^3} - \dfrac{1}{\mathrm{e}}$ (b) $\dfrac{1}{6\mathrm{e}^3} - \dfrac{1}{2\mathrm{e}}$ (c) $\dfrac{-\cosh(1)}{5\mathrm{e}^2} - \dfrac{2\sinh(1)}{5\mathrm{e}^2} + \dfrac{1}{2}$

(d) $\dfrac{1}{3\mathrm{e}^3} - \dfrac{1}{\mathrm{e}} - \dfrac{2}{3}$ (E) $\dfrac{1}{6\mathrm{e}^3} - \dfrac{1}{2\mathrm{e}} + \dfrac{1}{3}$

7. Find $\dfrac{dy}{dx}$ at $(1, \mathrm{e})$ if $(x+y)\ln y = 1 + \mathrm{e}$.

(a) -1 (b) 0 (C) $\dfrac{-\mathrm{e}}{1+2\mathrm{e}}$ (d) $\dfrac{\mathrm{e}}{1+2\mathrm{e}}$ (e) $\dfrac{\mathrm{e}}{1-2\mathrm{e}}$

8. Evaluate $\displaystyle\int 3x\cos(3x)\; dx.$

(a) $\sin(3x) + C$ (b) $\dfrac{1}{3}x\cos(3x) + \dfrac{1}{9}\cos(3x) + C$ (c) $x\sin(3x) - \dfrac{1}{9}\cos(3x) + C$

(d) $x\cos(3x) - \sin(3x) + C$ (E) $x\sin(3x) + \dfrac{1}{3}\cos(3x) + C$

9. $y = \dfrac{1}{2}\sin(2\arctan x)$ has the same graph as

(A) $y = \dfrac{x}{1+x^2}$ (b) $y = \sqrt{1+x^2}$ (c) $y = \dfrac{x}{\sqrt{1+x^2}}$

(d) $y = \dfrac{1}{2\sqrt{1+x^2}}$ (e) $y = \dfrac{1}{1+x^2}$

10. If $f(x) = \cos^{-1}(2x) = \arccos(2x)$, then $f'(x) =$

(a) $\dfrac{1}{\sqrt{4x^2-1}}$ (b) $\dfrac{-1}{\sqrt{1-4x^2}}$ (C) $\dfrac{-2}{\sqrt{1-4x^2}}$ (d) $\dfrac{2}{\sqrt{4x^2-1}}$ (e) $\dfrac{2}{\sqrt{1-4x^2}}$

Part II. WORK OUT PROBLEMS, PART CREDIT will be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicated your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

11. Find the area of the region bounded by the x-axis, the graph of $y = x\mathrm{e}^{-x}$, and the lines $x = 1$ and $x = 2$.

12. Find the partial fraction decomposition of $\dfrac{x^2+2}{x^2-x}$.

13. Evaluate $\displaystyle\int \frac{1}{x^3+x}\,dx.$

14. Use the Trapezoidal Rule to approximate the area of the lake behind the dam pictured at the right. The measurements are in hundreds of meters, made at intervals 50 meters apart.

15. An advertiser wants to place a billboard at some distance x from where viewers will stand at P. (See figure.) What choice of x will maximize the viewing angle θ? All distances are in feet. Hint: Use inverse trig functions to express θ as a function of x.

16. Compute $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^x$

17. Evaluate $\displaystyle\int_0^{3/2}\frac{x^2}{\sqrt{9-x^2}}\,dx.$

18. Evaluate $\displaystyle\lim_{x\to\infty}\frac{2}{x\ln x}\int_1^x \ln t\;dt.$

19. Let $f(x) = \mathrm{e}^{5x} + 3\mathrm{e}^{2x} + 2$

a) Compute $f(0)$.

b) Explain why f has an inverse function for all x.

c) Find the derivative of the inverse function at 6, i.e., find $(f^{-1})'(6)$.

20. The temperature of a cup of coffee as it is poured by a waiter is $180°$ F. The room temperature is $70°$ F. Three minutes later, the cup of coffee has cooled to $140°$ F. The temperature $T(°F)$ of the cup of coffee can be modelled by Newton's Law of Cooling as $\dfrac{dT}{dt} = k(T - 70)$. The solution to this differential equation is $T = 70 + C\mathrm{e}^{kt}$. If the customer prefers coffee at $120°$ F, how much time should she wait after it is poured before drinking it?