

MATH 152, Exam II, Version A
Fall, 1994

NAME

ID#

INSTRUCTOR'S NAME

SECTION #

INSTRUCTIONS

1. In Part I (problems 1-9), mark the correct choice on your SCANTRON sheet using a #2 pencil. Use the back of each page for scratch work. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned.
2. In Part II (problems 10-15), write all solutions in the space provided. Use the back of each page for scratch work, but all work to be graded must be shown in the space provided. **CLEARLY INDICATE YOUR FINAL ANSWER.**

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Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour. (6 points each)

1. Compute $\int_0^1 \frac{\ln x}{x} dx$.
- (a) e (b) -1 (c) -2 (d) 0 (E) divergent

2. Find the limit of the sequence $\left\{ \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \right\}$.
- (a) ∞ (b) 1 (c) 0 (D) -1 (e) $-\infty$

3. Which plot shows the direction field of the differential equation $\frac{dy}{dx} = \frac{y^2}{x^3}$?
- (A)
(b)
(c)
(d)
(e)

4. Set up an integral for the arclength of the curve $y = x^2$ from $(0, 0)$ to $(2, 4)$.

- (a) $\int_0^4 \sqrt{x^2 + y^2} dx$ (b) $\int_0^4 \sqrt{1 + x^2} dx$ (C) $\int_0^2 \sqrt{1 + 4x^2} dx$
 (d) $\int_0^2 \sqrt{1 + x^4} dx$ (e) $\int_0^4 (1 + 2x) dx$

5. The temperature (in $^{\circ}F$) at a time t hours after 9 AM, is given by the formula $T = 50 + 14 \sin\left(\frac{\pi t}{12}\right)$ for a typical day in December. What is the average temperature from 9 AM to 9 PM on such a day?

- (a) 78 (b) 50 (c) $50 + 14\pi$ (D) $50 + \frac{28}{\pi}$ (e) $50 + \frac{14}{\pi}$

6. For which values of C and D is the function $y = C + De^{3x}$ a solution of the different equation $\frac{dy}{dx} = 3y + 6$?

- (a) All C and D (b) $C = 3, D = -1$ only (C) $C = -2$, all D
 (d) $C = -2, D = 1$ only (e) $D = 0$, all C

7. The integral $\int_1^\infty \left(\frac{\sin x}{x}\right)^2 dx$ is

- (a) convergent by comparison to $1/x$.
- (B) convergent by comparison to $1/x^2$.
- (c) divergent by oscillation.
- (d) divergent to infinity.
- (e) divergent by comparison to $(\sin x)^2$.

8. Consider the differential equation for $0 \leq \theta \leq \frac{\pi}{2}$ given by $\frac{dy}{d\theta} - y \tan \theta = 1$. An integrating factor is

- (a) $\sec^2 \theta$ (b) 1 (c) $\tan \theta$ (d) $e^{\tan \theta}$ (E) $\cos \theta$

9. Find the limit of the sequence $a_n = \left(\sqrt{n+1} - \sqrt{n}\right) \sqrt{n + \frac{1}{2}}$

- (a) 0 (B) $\frac{1}{2}$ (c) 1 (d) 2 (e) ∞

Part II. WORK OUT PROBLEMS, PART CREDIT will be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicated your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

10. (8 points) Solve the initial value problem $\frac{dy}{dx} = y^2 + x^2y^2$, with $y(0) = 1$.

11. (7 points) A tank originally contains 100 gal of fresh water. Then water containing 1/2 lb of salt per gallon is poured into the tank at the rate of 2 gal/min, and the mixture is allowed to leave at the same rate. Let $Q(t)$ denote the number of pounds of salt in the tank at time t . Write a differential equation for $Q(t)$ and give the initial condition. (DO NOT SOLVE THE DIFFERENTIAL EQUATION.)

12. (8 points) A rubber ball is dropped from a height of 6 meters and bounces back to one third of its previous height after each fall, and continues to bounce until it comes to rest after an infinite numbers of bounces. Find the total distance it travels while bouncing. SUGGESTION: Draw a picture.

13. (8 points) Consider the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$.

(a) Write the series in summation notation, i.e. fill in the three blanks:

$$\sum_{n=\underline{\quad}}^{\underline{\quad}} \underline{\quad}$$

(b) Find the sum of the series.

14. (7 points) The curve $y = \frac{1}{2}x^2$ below $y = \frac{1}{2}$ is rotated about the y -axis. Find the surface area of the resulting bowl.

15. (8 points) Find the x -coordinate of the centroid of the area inside the triangle shown at the right.