MATH 152, Exam III, Version A
Fall, 1994

NAME

ID#

INSTRUCTOR'S NAME

SECTION #

INSTRUCTIONS

1. In Part I (problems 1 – 8), mark the correct choice on your SCANTRON sheet using a #2 pencil. Use the back of each page for scratch work. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned.

2. In Part II (problems 9 – 15), write all solutions in the space provided. Use the back of each page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWER.

MATH 152, Exam III, Version A

Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour. (6 points each)

1. Use the Taylor polynomial approximation $\sqrt{1+h} \approx 1 + \frac{1}{2}h$ to estimate $\int_0^1 \sqrt{1+x^4}\, dx$.

(a) 1.1 (b) 1.2 (c) 1.3 (d) 1.4 (e) 1.5

2. The parametric curve given by $x = t^2 - 4t$, $y = t^2 - 6t$ has a vertical tangent at

(a) $(-3, 3)$ (b) $(2, 3)$ (c) $(-3, -9)$ (d) $(-4, -8)$ (e) $(2, -8)$

3. Which ONE of the following series DIVERGES?

(a) $\sum_{n=1}^{\infty} \frac{1}{n(2n+1)}$ (b) $\sum_{n=1}^{\infty} \frac{n^{100}}{n!}$ (c) $\sum_{n=1}^{\infty} \frac{2n}{n+1}$

(d) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ (e) $\sum_{n=1}^{\infty} 3^{-n}$

4. Which of these plots represents the parametric curve
$x = 1 + \mathrm{e}^{-t/4}\cos t$, $y = \mathrm{e}^{-t/4}\sin t$ for the interval $0 \le t < \infty$?

(a)   (b)   (c)   (d)   (e)

5. The power series for $\dfrac{1}{1+2x^2}$ is $\displaystyle\sum_{n=0}^{\infty}(-1)^n\, 2^n\, x^{2n}$.

Find the radius of convergence.

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\sqrt{2}}$ (c) 1   (d) 2   (e) ∞

6. Find the Maclaurin series for e^{-x^2}.

(a) $1 - x^2 + \frac{1}{2!}x^4 - \frac{1}{3!}x^6 + \ldots$

(b) $1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots$

(c) $1 - x^2 + x^4 - x^6 + \ldots$

(d) $1 - 2xe^{-x^2} + \frac{1}{2!}(4x^2 - 2)e^{-x^2} + \ldots$

(e) $1 - x^2 e^{-x^2} + x^4 e^{-2x^2} - x^6 e^{-3x^2} + \ldots$

7. The coefficient of x^2 in the Maclaurin series for $\frac{1}{1-x} \cdot e^x$ is:

(a) $-\frac{3}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$ (e) $\frac{5}{2}$

8. Let $3 - \frac{1}{2}(x-1) + \frac{2}{5}(x-1)^2 + \frac{5}{9}(x-1)^3 + \ldots$ be the beginning of the Taylor series expansion of $f(x)$ about $x = 1$. Which of the following is true at $x = 1$ for the graph of $f(x)$?

(a) Increasing and concave up

(b) Increasing and concave down

(c) Decreasing and concave up

(d) Decreasing and concave down

(e) There is not enough information to determine which of these holds at $x = 1$.

Part II. WORK OUT PROBLEMS, PART CREDIT will be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicated your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

9. (8 points) Find the complete Taylor series expansion about $x = 1$ (not about zero) for $f(x) = x + x^2 + x^3$.

10. (8 points) Find the arclength of the parametrized curve
$x = 1 + e^{-t} \cos t$, $y = e^{-t} \sin t$ for the interval $0 \le t < \infty$.

11. (7 points) Determine whether or not the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n[\ln(n)]^2}$ converges, and justify your answer.

12. (7 points) The formula $I(\alpha) = 4\big(\alpha - \frac{1}{2}\sin(2\alpha)\big)$ gives the moment of inertia of a circular sector of radius 2 and central angle α about its bisector. Find the 3^{rd} degree Taylor polynomial about $\alpha = 0$ for $I(\alpha)$.

13. (7 points) The Maclaurin series for e^x is $f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots + \frac{1}{n!}x^n + \ldots$. If the series is truncated to end with the term $\frac{1}{n!}x^n$ and used to approximate e^x for x in $[0, \frac{1}{2}]$, what is the minimum value of n so that the error will be less than 0.01 for all x in $[0, \frac{1}{2}]$? Justify your claim that this many terms, but no fewer, are sufficient.

14. (7 points) Find the area between the x-axis and the parametric curve $x = t^2 + 1$, $y = t(2 - t)$, as shown below in the Maple plot generated by the Maple command: **plot([t^2 + 1, t*(2 - t), t=0..2]);**

15. (8 points) Let $g(x) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \ldots + \frac{1}{n}x^n + \ldots$.

(a) Find the series expansion of $g'(x)$.

(b) The series expansion of $g'(x)$ happens to be a geometric series.
Find the sum of this geometric series for $g'(x)$.

(c) Find the exact value of $g\left(\frac{1}{2}\right)$. Hint: Integrate $g'(x)$.