MATH 152

Fall 1995

Exam 2

Test Form A

NAME ______________________________ ____________________

LAST FIRST

ID#

INSTRUCTOR'S NAME

SECTION #

INSTRUCTIONS

1. In Part I (Problems 1 — 12), mark the correct choice on your SCANTRON sheet using a #2 pencil. Use the back of each page for scratch work. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned.

2. In Part II (Problems W1 – W6), write all solutions in the space provided. Use the back of each page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWER.

Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour.

(Problems 1 – 7: 6 points each) (Problems 8 – 12: 3 points each)

1. (6 points) A sequence $\{a_n\}$ is given by $a_1 = 2$ and $a_{n+1} = 3 + \frac{1}{4}a_n^2$. Then a_3 is

(a) 6 (b) $\frac{53}{4}$ (c) $\frac{61}{4}$ (d) 7 (e) 4

2. (6 points) Let y be the function which is the solution to the initial value problem $y' = xy^2 + x - y$ with $y(1) = \frac{1}{2}$. Then an equation for the tangent line to the graph of y at the point where $x = 1$ is

(a) $y = 7x - 5$ (b) $4y - 3x = -1$ (c) $y = x - \frac{1}{2}$

(d) $y = \frac{1}{2}$ (e) $8y - 6x = -2$

3. (6 points) Find the sum of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}2^{3n+1}}{3^{2n}}$.

(a) 16/17 (b) $-16/17$ (c) 16 (d) -16 (e) $-18/17$

4. (6 points) The linear density in a rod 15 meters long is $\dfrac{1}{\sqrt{x+1}}\,\dfrac{\text{kg}}{\text{m}}$, where x is measured in meters from one end of the rod. Find the average density of the rod.

(a) $\dfrac{3}{15}\,\dfrac{\text{kg}}{\text{m}}$ (b) $\dfrac{5}{8}\,\dfrac{\text{kg}}{\text{m}}$ (c) $\dfrac{6}{15}\,\dfrac{\text{kg}}{\text{m}}$ (d) $\sqrt{\dfrac{2}{17}}\,\dfrac{\text{kg}}{\text{m}}$ (e) $3\,\dfrac{\text{kg}}{\text{m}}$

5. (6 points) Which of the following is the direction field for the differential equation $y' = \dfrac{x\,y}{25}$?

(a)

(b)

(c)

(d)

(e)

6. (6 points) The limit of the sequence $\left\{\arctan\left(\dfrac{n}{n+1}\right)\right\}$ is

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) 1 (e) ∞

7. (6 points) $\lim_{n\to\infty}\left(\sqrt{n^4+n^2}-n^2\right) =$

(a) 0 (b) 1 (c) n (d) 1/2 (e) ∞

8. (3 points) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ must converge.

(a) True (b) False

9. (3 points) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

(a) True (b) False

10. (3 points) $\sum_{n=1}^{\infty} \frac{1}{2} r^n$ converges for all r.

(a) True (b) False

11. (3 points) The series $\sum_{n=1}^{\infty} \frac{1}{2n}$

(a) converges. (b) diverges.

12. (3 points) The series $\sum_{n=1}^{\infty} (-1)^n$

(a) converges. (b) diverges.

Part II. WORK OUT PROBLEMS, PART CREDIT will be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicate your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

W1. (7 points) If $y(x)$ solves $y' = xy$ and $y(1) = 2$, then find $y(2)$.

W2. (8 points) Find the x-coordinate of the centroid of the region bounded by the parabola $x = y^2$ and the line $x = 4$.

W3. (5 points) Consider the infinite series $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$.

(i) Find a formula for the n^{th} partial sum, $\displaystyle S_n = \sum_{k=1}^{n} \ln\left(\frac{k}{k+1}\right)$, of the series.

(ii) Does the infinite series converge? If it converges, find its sum. Justify your answer.

W4. (5 points) At time $t = 0$ a tank contains 1 lb. of salt dissolved in 100 gal. of water. Assume that water containing $\frac{1}{4}$ lb. of salt per gallon is entering the tank at a rate of 3 gal/min, and that the well-stirred solution is leaving the tank at the same rate. Find the amount of salt, $Q(t)$, in the tank at time t.

W5. (5 points) A window consists of two parallel panes of tinted glass with a small air space between the panes. Whenever light hits either pane of glass on either side, 1/3 of the light passes through the pane and 2/3 is reflected back. If a light shines on one side of the window, what fraction of the light passes through the window?

W6. (6 points) Solve: $4xy' + 2y = e^{x^2}$

W7. (7 points) Find the length of the curve $y = x^{3/2}$ from $x = 1$ to $x = 2$.