MATH 152

Fall 1995

Exam 3

Test Form A

NAME ____________________________ ____________________
LAST FIRST

ID#

INSTRUCTOR'S NAME

SECTION #

INSTRUCTIONS

1. In Part I (Problems 1—11), mark the correct choice on your SCANTRON sheet using a #2 pencil. Use the back of each page for scratch work. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned.

2. In Part II (Problems 12–18), write all solutions in the space provided. Use the back of each page for scratch work, but all work to be graded must be shown in the space provided. CLEARLY INDICATE YOUR FINAL ANSWER.

Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour.

(5 points each)

1. Find an equation of the tangent line to the parametric curve $x = 3t^2 - 2t$, $y = 2t^3 + t$ at the point where $t = 2$.

(a) $y = \frac{2}{5}x$ (b) $y = (6t^2 + 1)(x - 8) + 18$ (c) $y = \frac{2}{5}x + \frac{74}{5}$

(d) $y = \frac{5}{2}x - 2$ (e) $y = 25x - 152$

2. The series $\sum_{n=1}^{\infty}(-1)^n \frac{3n}{n^2+1}$ is

(a) absolutely convergent. (b) conditionally convergent.

(c) divergent. (d) conditionally divergent.

3. The series $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ is

(a) convergent by the integral test. (b) convergent by the ratio test.

(c) convergent by the comparison test. (d) convergent by the limit comparison test.

(e) divergent.

4. Find the radius of convergence of the Maclaurin series for e^{-x}.

(a) $-\infty$ (b) 0 (c) 1 (d) e (e) ∞

5. Which of the following plots is the graph of the parametric equations

$$x = \sin^2(t), \quad y = \sin^4(t), \qquad \text{for} \quad -\infty \le t \le \infty?$$

(a) (b) (c)

(d) (e)

6. If the series $\sum_{n=1}^{\infty} a_n(x-6)^n$ has radius of convergence $r = 3$, then at $x = 2$ the series

(a) converges absolutely. (b) converges conditionally. (c) diverges.
(d) There is insufficient information to determine convergence or divergence.

7. If $f(x)$ is the function whose Maclaurin series is
$f(x) = \dfrac{x^2}{2} - \dfrac{x^3}{3} + \dfrac{x^4}{4} - \ldots + (-1)^n \dfrac{x^n}{n} + \ldots,$ compute $f'\left(\dfrac{1}{2}\right)$.

(a) $-\dfrac{2}{3}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{2}{3}$ (d) $e^{1/2}$ (e) 2

8. The series $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{n!}+\ldots$ converges to e.

So the series $\frac{3}{1!}+\frac{3}{2!}+\frac{3}{3!}+\frac{3}{4!}+\ldots+\frac{3}{(n+1)!}+\ldots$ must converge to the limit

(a) $3e-3$ (b) $3e-1$ (c) $3e$ (d) $3e+1$ (e) $3e+3$

9. Let $f(x)$ be the function whose Taylor series at 3 is $f(x)=\sum_{n=1}^{\infty}\frac{1+\sqrt{n}}{(n+1)!}(x-3)^n$.

Find $f^{(16)}(3)$, i.e., find the 16-th derivative of f at 3.

(a) $\frac{5}{17!}$ (b) $\frac{5}{16!}$ (c) $\frac{5}{17}$ (d) 5 (e) $5(3)^{16}$

10. In the Maclaurin series for $\frac{\sin(x)}{x}$, what is the coefficient of x^4?

(a) $-\frac{1}{4!}$ (b) $-\frac{1}{5!}$ (c) 0 (d) $\frac{1}{5!}$ (e) $\frac{1}{4!}$

11. Find the center, c, and radius of convergence, r, of the series $\sum_{n=1}^{\infty}\frac{n(x+2)^n}{5^{n-1}}$.

(a) $c=2, r=5$ (b) $c=2, r=\frac{1}{5}$ (c) $c=-2, r=5$

(d) $c=-2, r=\frac{1}{5}$ (e) $c=-\frac{2}{5}, r=1$

Part II. WORK OUT PROBLEMS, PART CREDIT will be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicate your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded.

12. (5 points) Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{2^n}$ converges or diverges.

13. (5 points) Determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ converges or diverges.

14. (5 points) Determine if the series $\displaystyle\sum_{n=1}^{\infty} n\, e^{-n^2}$ converges or diverges.

15. (8 points)

(a) Find the Maclaurin series of the function $f(x) = \sin(x^2)$. Give at least 4 non-zero terms or the general term.

(b) Use this series for $\sin(x^2)$ to approximate $\int_0^{0.1} \sin(x^2)\, dx$ to within $\pm 10^{-10}$. Justify the number of terms you use.

16. (5 points) Find parametric equations for the line segment from the point $(2, 3)$ to the point $(2, 5)$. Be sure to give the domain of the parameter.

17. (9 points)
(a) Find the total length of the astroid $x = \cos^3(\theta)$, $y = \sin^3(\theta)$.

(b) Find the surface area obtained by rotating the astroid about the x-axis.

18. (8 points)

(a) Find the Taylor polynomial of degree 2 for the function $f(x) = \ln(x)$ at $c = 3$.

(b) Give the remainder term.

(c) Use the remainder term to estimate the maximum possible error when the polynomial is used to approximate $\ln(x)$ on the interval $[2, 4]$.