

**MATH 152**

**Spring 1997**

**Exam 1 — 200 points**

**Test Form A**

**NAME**

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LAST

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FIRST

**ID#**

**INSTRUCTOR'S NAME**

**SECTION #**

**INSTRUCTIONS**

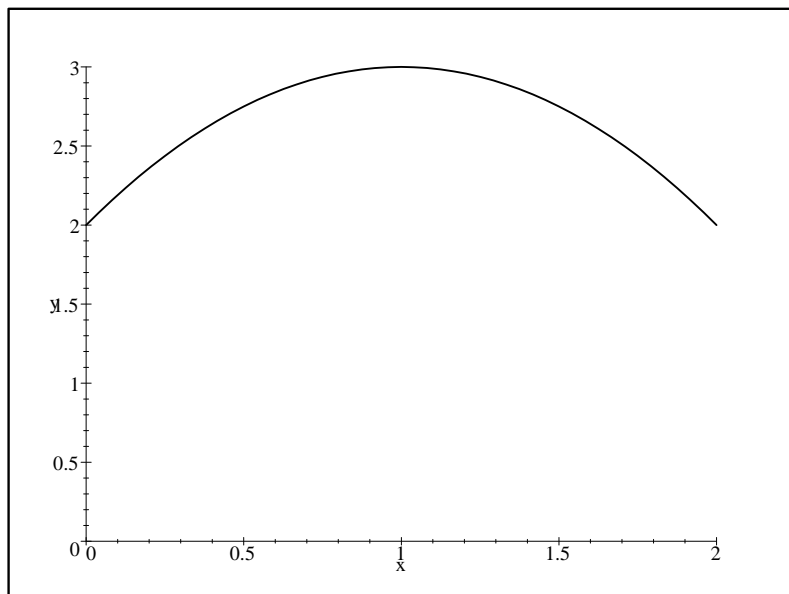
1. In Part I (Problems 1–10), mark the correct choice on your SCANTRON sheet using a #2 pencil. Use the back of each page for scratch work. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned. Calculators may not be used for this part.
2. In Part II (Problems 11–15), write all solutions in the space provided. Use the back of each page for scratch work, but all work to be graded must be shown in the space provided. **CLEARLY INDICATE YOUR FINAL ANSWER.** Calculators may be used for this part.

**Part I.** MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour. Each question is worth 12 points.

1. Compute  $\int x \sin x \, dx$ .  
a)  $-\frac{x^2}{2} \cos(x) + C$     b)  $x \sin x - \cos x + C$     c)  $\frac{x^2}{2} \sin x + C$   
d)  $-x \cos x + \sin x + C$     e)  $-x \cos x + C$
2. Compute  $\int_0^1 x e^{x^2} \, dx$   
a)  $\frac{1}{2}(e^{1/3} - 1)$     b)  $\frac{1}{2}(e - 1)$     c)  $e - 1$     d)  $1$     e)  $e^2 - 1$
3. Compute  $\int_1^\infty \frac{x}{(1+x^2)^2} \, dx$   
a)  $\frac{1}{4}$     b)  $\frac{\ln(2)}{2}$     c)  $1/2$     d)  $0$     e) does not converge
4. Which integral calculates the area between  $y = 2x + 1$  and  $y = 7 + x - x^2$ ?  
a)  $\int_{-3}^2 (2x + 1)(7 + x - x^2) \, dx$     b)  $\int_{-3}^2 x^2 + x - 6 \, dx$     c)  $\int_{-3}^2 6 - x - x^2 \, dx$   
d)  $\int_{-3}^2 \frac{2x + 1}{7 + x - x^2} \, dx$     e)  $\int_{-3}^2 8 + 3x - x^2 \, dx$
5. Which integral calculates the area between the curves  $y = x - 1$  and  $x + 1 = y^2$ ?  
a)  $\int_{-2}^1 (x - 1) - (x + 1) \, dx$     b)  $\int_{-2}^1 (x + 1) - (x - 1) \, dx$     c)  $\int_{-1}^2 \sqrt{x + 1} - (x - 1) \, dx$   
d)  $\int_{-1}^2 (y^2 - 1) - (y + 1) \, dy$     e)  $\int_{-1}^2 (y + 1) - (y^2 - 1) \, dy$
6. Which integral computes the volume of the solid generated by revolving the region between the curves  $y = 2x$  and  $y = x^2$  about the  $y$ -axis?  
a)  $\int_0^2 2\pi x(2x - x^2) \, dx$     b)  $\int_0^2 \pi(2x - x^2)^2 \, dx$     c)  $\int_0^2 \pi(2x)^2 - \pi(x^4) \, dx$   
d)  $\int_0^4 2\pi y(\sqrt{y} - y/2) \, dy$     e)  $\int_0^4 \pi(\sqrt{y} - y/2)^2 \, dy$
7. Compute  $\int \frac{dx}{x^3 - x^2}$ .  
a)  $\ln \sqrt{\frac{|x+1|}{|x-1|}} - \ln|x| + C$     b)  $\ln \left( \frac{|x-1|}{|x|} \right) + C$     c)  $\ln \left( \frac{|x-1|}{|x|} \right) + \frac{1}{x} + C$   
d)  $\ln \sqrt{\frac{|x-1|}{|x+1|}} + C$     e)  $\frac{1}{3} \ln|x^3 - x| + C$
8. The substitution  $x = 2 \tan t$  transforms the integral  $\int_0^2 \frac{dx}{\sqrt{x^2 + 4}}$  into  
a)  $\int_0^{\pi/4} \sec t \, dt$     b)  $\int_0^2 \sec t \, dt$     c)  $\int_0^2 \frac{1}{\sec t} \, dt$   
d)  $\int_0^{\pi/4} \frac{1}{\sec t} \, dt$     e)  $\int_0^{\pi/4} \sec t \tan t \, dt$

9. The graph of a function  $f$  is given below. Which of the following statements most accurately describes the relationship between  $\int_0^2 f(x) dx$  and its approximation by  $T_4$ , the trapezoidal rule with 4 subdivisions.



- a)  $T_4$  is greater than  $\int_0^2 f(x) dx$     b)  $T_4$  is less than  $\int_0^2 f(x) dx$     c)  $T_4$  is equal to  $\int_0^2 f(x) dx$   
d) Not enough information to determine    e) None of these
10. Which statement most accurately describes the convergence or divergence of  $\int_1^\infty \frac{x dx}{\sqrt{x^5 + 1}}$ ?

- a) The integral converges because  $\frac{x dx}{\sqrt{x^5 + 1}} \leq \frac{1}{x^{5/2}}$  and  $\int_1^\infty \frac{dx}{x^{5/2}}$  converges.  
b) The integral diverges because  $\frac{x dx}{\sqrt{x^5 + 1}} \geq \frac{1}{x^{3/2}}$  and  $\int_1^\infty \frac{dx}{x^{3/2}} = \infty$ .  
c) The integral converges because  $\frac{x dx}{\sqrt{x^5 + 1}} \leq \frac{1}{x^4}$  and  $\int_1^\infty \frac{dx}{x^4}$  converges.  
d) The integral diverges because  $\frac{x dx}{\sqrt{x^5 + 1}} \geq \frac{1}{x^4}$  and  $\int_1^\infty \frac{dx}{x^4} = \infty$ .  
e) The integral converges because  $\frac{x dx}{\sqrt{x^5 + 1}} \leq \frac{1}{x^{3/2}}$  and  $\int_1^\infty \frac{dx}{x^{3/2}}$  converges.

**Part II.** WORK OUT PROBLEMS, PART CREDIT may be given.

CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicate your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded. Clearly indicate your final answer. Each question is worth 16 points.

11. Compute  $\int \cos^3 x \, dx$ .

12. Compute  $\int x^2 e^{-x} \, dx$ .

13. Compute  $\int \sqrt{9 - x^2} \, dx$ .
14. Set up the integral required to compute the volume of the region whose base is a unit circle in the  $x, y$  plane and whose cross sections perpendicular to the  $x$ -axis are equilateral triangles. You do not need to compute this integral.

15. The error in approximating  $\int_a^b f(x) dx$  by using trapezoidal rule is no bigger than  $\frac{K(b-a)^3}{12n^2}$  where  $K$  is the maximum value of  $|f''|$  over the interval  $a \leq x \leq b$  and  $n$  is the number of subdivisions. Find the minimum number of subdivisions ( $n$ ) that will ensure that the error in computing  $\int_2^4 \ln x dx$  by trapezoidal rule is less than  $1/400=0.0025$ .