MATH 152

Spring 1997

Exam III — 202 points

Test Form A

NAME _____ _____

LAST FIRST

ID#

INSTRUCTOR'S NAME

SECTION #

INSTRUCTIONS

1. In Part I (Problems 1–10), mark the correct choice on your SCANTRON sheet using a #2 pencil. For your own records, record your responses on your exam (which will be returned to you). The SCANTRON will be collected after 1 hour and will not be returned. Calculators may not be used for this part.

2. In Part II (Problems 11–16), write all solutions in the space provided. CLEARLY INDICATE YOUR FINAL ANSWER. Calculators may be used for this part.

Note: The following formula for the curvature of the path $\mathbf{R}(t)$ may be useful

$$k = \frac{|\mathbf{R}'(t) \times \mathbf{R}''(t)|}{|\mathbf{R}'(t)|^3}$$

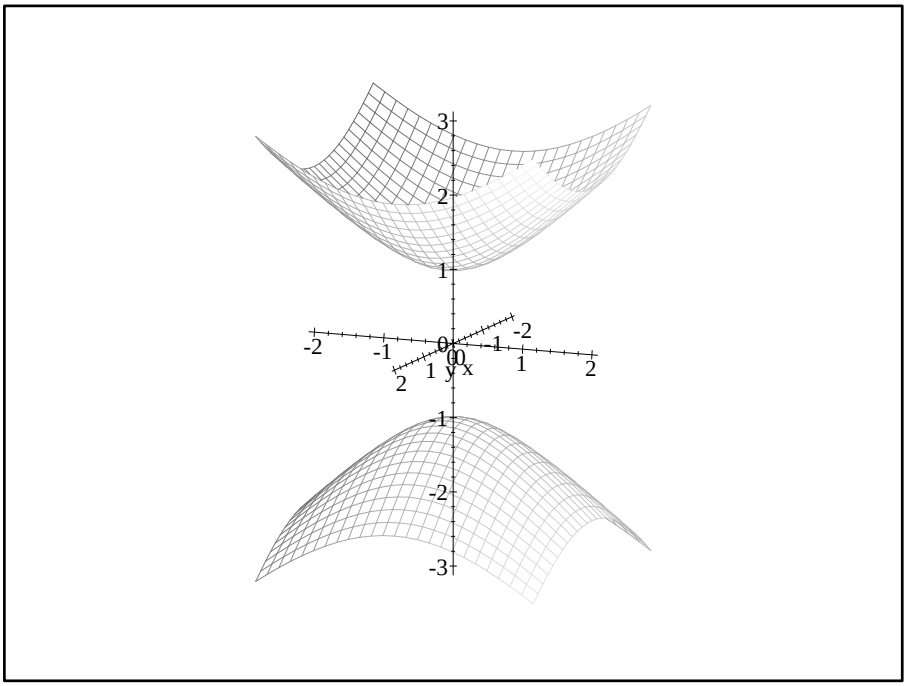
Part I. MULTIPLE CHOICE, NO PART CREDIT, NO CALCULATORS

The SCANTRON forms will be collected at the end of 1 hour. Each question is worth 10 points.

1. If $\mathbf{A}$ and $\mathbf{B}$ are vectors in R^3, which one of the following expressions has no meaning?
   a) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ b) $(\mathbf{A} \cdot \mathbf{B}) \times \mathbf{C}$ c) $(\mathbf{A} \cdot \mathbf{B})\mathbf{C}$ d) $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$ e) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$

2. Find the scalar projection of the vector $\langle 1, -7, 2\rangle$ onto the vector $\langle 3, -2, -1\rangle$.
   a) $\frac{15}{\sqrt{54}}$ b) 15 c) $\sqrt{14}$ d) $\frac{15}{\sqrt{14}}$ e) $\sqrt{54}$

3. Which of the following equations generates the plot given below?
   a) $x^2 - z^2 - y^2 = 1$ b) $y^2 - z^2 - x^2 = 1$ c) $z^2 - x^2 - y^2 = 1$
   d) $z^2 - x^2 + y^2 = 1$ e) $z^2 - y^2 + x^2 = 1$



4. Find the equation of the tangent line to the curve $x = t^2$, $y = t^3 - 4t$, $z = 4/t$ at the point $(4, 0, 2)$.

a) $x = 4t + 4$, $y = 8t$, $z = 2 - t$ b) $x = 4t + 4$, $y = 8t + 4$, $z = -t + 1$

c) $x = 4t + 4$, $y = 4t + 4$, $z = t + 1$ d) $x = 4t + 4$, $y = 12t + 4$, $z = 2t + 1$

e) $x = 2t + 4$, $y = 3t^2 - 4$, $z = -4/t^2 + 3$

5. Suppose a particle is moving with constant speed along the curve given below in the direction indicated. When the particle reaches the point P, which direction is its acceleration vector pointing?

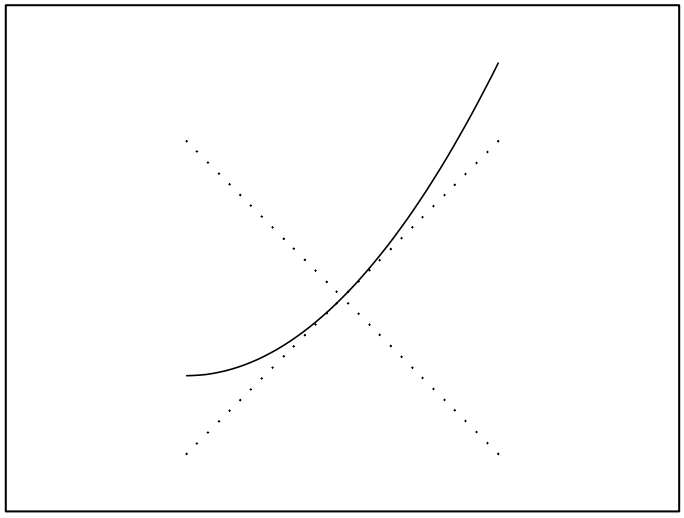
a) toward quadrant I

b) toward quadrant II

c) toward the direction T (the tangent)

d) toward the direction N (the normal)

e) the acceleration vector is zero



6. Describe the level curves of the function $f(x, y) = \dfrac{-2y}{x^2 + y^2 + 1}$.

a) spheres b) ellipses c) circles d) hyperbolas e) parabolas

7. Compute $\dfrac{\partial}{\partial y}\{y \sin(xy^2)\}$.

a) $\cos(2xy)$ b) $y \cos(y^2)$ c) $y^3 \cos(xy^2)$

d) $\sin(xy^2) + y \cos(2xy)$ e) $\sin(xy^2) + 2xy^2 \cos(xy^2)$

8. Find the equation of the tangent plane to the surface $z = x^2 - y^2$ at the point $x = 2$, $y = 1$.

a) $z = 2x - 2y + 1$ b) $z = 2x(x-2) - 2y(y-1) + 3$ c) $z = 2y(x-2) - 2x(y-1) + 3$

d) $z = 2x - 4y + 3$ e) $z = 4x - 2y - 3$

9. Which one of the following statements most accurately applies to the two lines given by the following equations $x = t,\ y = 6t,\ z = -t$ and $x = 2s - 4,\ y = s - 2,\ z = 8 - 4s$.

a) the two lines intersect and are perpendicular to each other

b) the two lines intersect but are not perpendicular

c) the two lines do NOT intersect and are skew

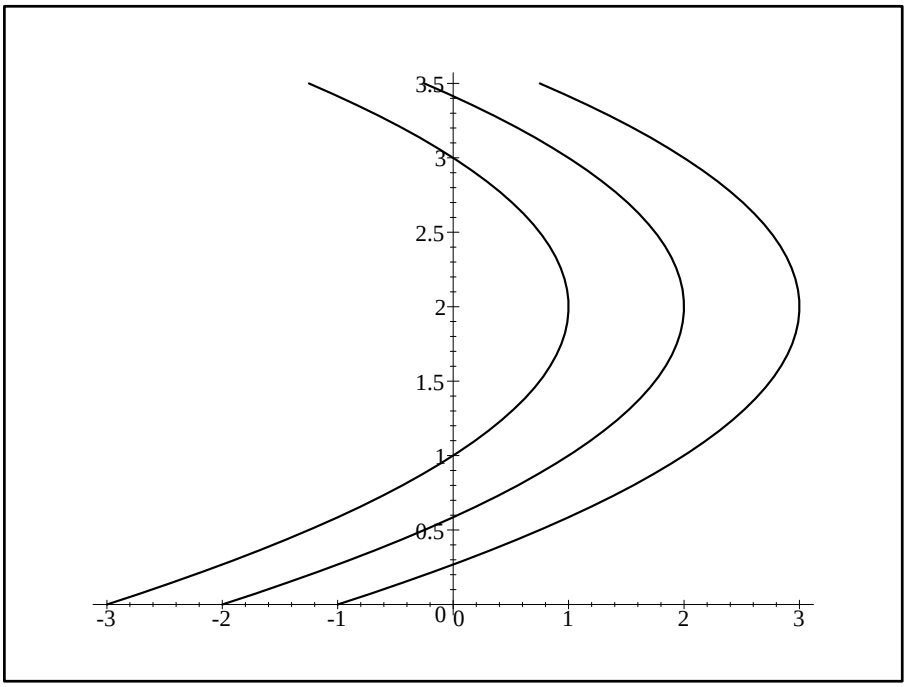
d) the two lines do NOT intersect and are parallel

e) not enough information to determine direction

10. The diagram below contains the plots of three level curves of a function $f : \mathbf{R}^2 \to \mathbf{R}$. From this information give an approximate value for $f_x(2,2)$ and $f_y(2,2)$.

a) $f_x(2,2) = -2$ and $f_y(2,2) = 0$ b) $f_x(2,2) = 0$ and $f_y(2,2) = -2$ c) $f_x(2,2) = 3$ and $f_y(2,2) = 0$

d) $f_x(2,2) = 0$ and $f_y(2,2) = 3$ e) $f_x(2,2) = -1$ and $f_y(2,2) = 0$



Part II. WORK OUT PROBLEMS, PART CREDIT may be given. CALCULATORS ARE PERMITTED after the SCANTRONS are collected.

Show all relevant steps in your solution. Clearly indicate your answer. Unsupported answers will not be given credit. Only work shown in the space provided will be graded. Clearly indicate your final answer. Each question is worth 17 points.

11. Find the angle between the diagonal of a cube of side length 1 and the diagonal of one of its faces.

12. Find the equation of the plane containing the point $(1, 1, 1)$ and the line $x = 2t$, $y = 2 - 3t$, $z = -2t - 1$.

13. Find the distance from the point $(2, 1, 3)$ to the plane $x - 2y + z = 1$.

14. Find the equation of the osculating circle to the curve $y = 2x^2$, at the point (0,0).

15. The acceleration of a particle is $\langle 2\cos t, 8\sin t\rangle$ for all t. It's initial (at $t = 0$) velocity vector is $\langle 2, -2\rangle$ and its initial position vector is $\langle 1, 1\rangle$. Find the position of the particle at any future time t. Carefully derive any formulas that you use.

16. Compute $\lim_{(x,y)\mapsto(0,0)} \dfrac{x^3 y}{x^4 + y^4}$, if it exists. Give reasons for your answer.