MATH 253.504–506

Exam 2, version A

10/27/99

Notice that the only problems in which you are asked to evaluate an integral are #1 and #7.

1. (15 pts.) Find $\bar{y}$ of the center of mass of the region $1 \leq x^2 + y^2 \leq 9$, $y \geq 0$, if the density is constant.

2. (15 pts.) Use Lagrange multipliers to determine the maximum and minimum values of $x + 4y$ on the ellipse $x^2 + 2y^2 = 1$, and give the points where these occur.

3. (10 pts.) Convert $\int_0^4 \int_0^x \sqrt{x^2 + y^2} \, dy \, dx$ to an integral in polar coordinates, but don’t evaluate it.

4. (15 pts.) Reverse the order of integration to write $\int_0^3 \int_{(x-1)^2}^{x+1} f(x, y) \, dy \, dx$ as the sum of 1 or more integrals in the order $dx \, dy$.

5. (15 pts.) Suppose that $E$ is the region in space bounded above by the sphere $(x - 2)^2 + (y + 1)^2 + z^2 = 25$ and below by the plane $z = 3$. Set up, but do not evaluate $\iiint_E xz \, dV$, in the order $dz \, dy \, dx$.

6. (15 pts.) Determine the maximum and minimum of $f(x, y) = xy + 3x$ on the closed region bounded above by $y = 9 - x^2$ and below by the $x$ axis, and give the points where they occur.

7. (15 pts.) Find the area of the part of the surface $z = x + y^2$ which lies above the triangle in the $x, y$ plane with vertices $(0, 0), (0, 2), \text{ and } (2, 2)$. (Hint: if you run into a hard integral, you’ve missed something.)