# Applied Analysis Qualifier Syllabus

## Function spaces and operators

- Hilbert spaces, Banach spaces, and dual spaces
- Spaces of functions – $L^p$, $\ell^p$, C[a,b], $C^k[a,b]$, $H^k[a,b]$ (Sobolev space)
- Orthogonal expansions – Fourier series, orthogonal polynomials
- Operators – bounded operators, compact operators, integral equations, Sturm-Liouville eigenvalue problems, spectral theory
- Fourier transforms
- Distributions

## Approximation analysis

- Polynomials – Weierstrass approximation theorem, least squares with orthogonal polynomials
- Splines – linear, quadratic, cubic
- Approximation with Fourier series – pointwise, uniform, and mean convergence
- Asymptotic analysis – big "oh" and little "oh" notation, asymptotic formulas, Laplace's method, Watson's lemma.

## Calculus of variations

- Variational (Fréchet/Gâteaux) derivative of nonlinear functionals
- Euler-Lagrange equations; natural boundary conditions; several independent variables
- Lagrangians and Hamiltonians
- Minimax principle and estimating eigenvalues

## References


1. N. G. de Bruijn, *Asymptotic Methods in Analysis*, Dover Publications, New York, 1981.
2. E. W. Cheney, *Introduction to approximation theory.* Reprint of the second (1982) edition. AMS Chelsea Publishing, Providence, RI, 1998.
3. G. B. Folland, *Fourier Analysis and Its Applications*, Wadsworth & Brooks/Cole, Pacific Grove, CA, 1992.
4. I. M. Gelfand and S. V. Fomin, *Calculus of Variations*, Prentice-Hall, Englewood cliffs, NJ, 1963.
5. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation*, Perseus books, Reading, MA, 1995.
6. M. Reed and B. Simon, *Methods of Mathematical Physics. I. Functional Analysis* 2nd edition, Academic Press, New York, 1980.
7. F. Riesz and B. Sz.-Nagy, *Functional Analysis*, Ungar Publishing, New York, 1955.
8. G. P. Tolstov, *Fourier Series*, Dover Publications, New York, 1976.


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# PROPOSED CONTENT OF THE QUALIFYING EXAM ON APPL. MATH/NUMERICAL ANALYSIS. Part II. Numerical Analysis

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## Content



## 1. Finite element method

(Johnson, Ciarlet, Strang & Fix, Ern & Guermond, Grossmann et al)

(1) Weak (variational) formulation of second order elliptic problems and characterization of the energy space: essential and natural boundary condition.
(2) Ritz-Galerkin method and finite element method.
(3) Finite element spaces of piece-wise linear and quadratic polynomials (over triangles and tetrahedra) and piece-wise bilinear and biquadratic polynomials over rectangles.
(4) Error estimates, Bramble-Hilbert lemma, Nitsche trick. Strans's Lemmas.
(5) Variational "crimes": nonconforming spaces, and approximation of the bilinear and linear forms by quadrature rules.
(6) Galerkin finite element method for transient problems.

## 2. Numerical methods for parabolic problems

(Johnson, Larsen &Thomee)

(1) Finite difference approximations in time: explicit, implicit and Crank-Nicolson schemes, multistep methods, Runge-Kutta methods.
(2) Stability: maximum principle, Fourier mode analysis, matrix stability and energy type estimates (Courant condition).
(3) Error estimates for finite element and finite difference approximations.

## 3. References

(1) S. Larsen and V. Thomee, Partial Differential Equations with Numerical Methods, Springer-Verlag, Texts in Applied Mathematics 45, 2003
(2) C. Johnson, Numerical Solutions of PDE's by the Finite Element Method, Cambridge University Press, 1987.
(3) C. Grossmann, H.-G. Roos, and M. Stynes, Numerical Treatment of PDEs, Springer, 2007.
(4) A. Ern and J-L Guermond, Theory and Practice of Finite Elements, Springer, 2004.
(5) Ph. Ciarlet, The Finite Element Method for Elliptic Problems, SIAM, 2002.
(6) G. Strang and G. Fix, An Analysis of the Finite Element Method, Prentice Hall, Englewood Cliffs, N.J., 1973.