Applied Analysis Qualifier Syllabus

Function spaces and operators

- Hilbert spaces, Banach spaces, and dual spaces
- Spaces of functions – $L^p$, $C[a,b]$, $C^k[a,b]$, $H^k[a,b]$ (Sobolev space)
- Orthogonal expansions – Fourier series, orthogonal polynomials
- Operators – bounded operators, compact operators, integral equations, Sturm-Liouville eigenvalue problems, spectral theory
- Fourier transforms
- Distributions

Approximation analysis

- Polynomials – Weierstrass approximation theorem, least squares with orthogonal polynomials
- Splines – linear, quadratic, cubic
- Approximation with Fourier series – pointwise, uniform, and mean convergence

Calculus of variations

- Variational (Fréchet/Gâteaux) derivative of nonlinear functionals
- Euler-Lagrange equations; natural boundary conditions; several independent variables
- Lagrangians and Hamiltonians
- Minimax principle and estimating eigenvalues

References


Updated 8/17/08 (fjn).
Content

1. Finite element method
   (Johnson, Ciarlet, Strang & Fix, Ern & Guermond, Grossmann et al)
   (1) Weak (variational) formulation of second order elliptic problems and characterization of the energy space: essential and natural boundary condition.
   (2) Ritz-Galerkin method and finite element method.
   (3) Finite element spaces of piece-wise linear and quadratic polynomials (over triangles and tetrahedra) and piece-wise bilinear and biquadratic polynomials over rectangles.
   (4) Error estimates, Bramble-Hilbert lemma, Nitsche trick, Strang’s Lemmas.
   (5) Variational "crimes": nonconforming spaces, and approximation of the bilinear and linear forms by quadrature rules.
   (6) Galerkin finite element method for transient problems.

2. Numerical methods for parabolic problems
   (Johnson, Larsen & Thomee)
   (1) Finite difference approximations in time: explicit, implicit and Crank-Nicolson schemes, multistep methods, Runge-Kutta methods.
   (2) Stability: maximum principle, Fourier mode analysis, matrix stability and energy type estimates (Courant condition).
   (3) Error estimates for finite element and finite difference approximations.

3. References