1. Let \( u : \mathbb{C} \to \mathbb{R} \) be a continuous function such that \( e^u \) is a harmonic function. Prove that \( u \) must be a constant function.

2. Suppose that \( f \) is holomorphic in \( \{ z \in \mathbb{C} : 0 < |z| < 1 \} \), the punctured unit disk. Prove that the point 0 is a removable singularity for the function \( f \) if and only if the point 0 is a removable singularity for the function \( f'f'' \) (the product of the first derivative of \( f \) and the second derivative of \( f \)).

3. Show that
\[
\int_0^{2\pi} \frac{\cos(\theta)}{1 - \cos(\theta) + \frac{1}{4}} \, d\theta = \frac{4\pi}{3}.
\]

4. Suppose that \( p(z) \) and \( q(z) \) are polynomials of degree 2013, and all the zeroes of \( p \) lie inside the open unit disk. Prove that if \( |q(z)| \leq |p(z)| \) whenever \( |z| = 1 \), then \( |q(z)| \leq |p(z)| \) whenever \( |z| > 1 \).

5. Does there exist a sequence \( \{ p_n(z) \}_{n=1}^\infty \) of polynomials such that \( \lim_{n \to \infty} p_n(z) = 0 \) when \( z = 0 \) and \( \lim_{n \to \infty} p_n(z) = 1 \) when \( z \neq 0 \)? Explain why or why not.

6. Suppose \( f \) is an entire function that is odd [antisymmetric: that is, \( f(z) = -f(-z) \) for every \( z \)]. Show that \( f(\mathbb{C}) \), the range of \( f \), is either \( \mathbb{C} \) or \( \{0\} \).

7. Suppose \( \Omega = \mathbb{C} \setminus \{ z : \text{Re}(z) = 0 \text{ and } -1 \leq \text{Im}(z) \leq 1 \} \) (the complex plane with a slit along the imaginary axis from \(-i\) to \(i\)). There exists a holomorphic function \( f \) on \( \Omega \) with the property that \( (f(z))^2 = z^2 + 1 \) for every \( z \) in \( \Omega \). Prove that \( f \) is odd: namely, \( f(z) = -f(-z) \) for every \( z \) in \( \Omega \).

8. Show that the function sending \( z \) to \( i \tan(iz) \) is a biholomorphic mapping from the horizontal strip \( \{ z \in \mathbb{C} : -\pi/2 < \text{Im}(z) < \pi/2 \} \) onto the complex plane with slits along the real axis from \(-\infty\) to \(-1\) and from 1 to \(\infty\).

9. Does there exist an entire function that takes every complex value on every line through the origin? Explain why or why not.

10. State one of the following theorems and sketch the proof: the Riemann mapping theorem, Mittag-Leffler’s theorem, or Morera’s theorem.