1. Give the statements of the following results:
   (a) Montel’s theorem;
   (b) Harnack’s lemma;
   (c) Mittag-Leffler’s theorem.

2. Let \( f(z) \) be analytic in \( \Omega = \{ |z| > 1 \} \). Suppose that \( f \) satisfies \( |f(z)| < |z|^n \) for all \( z \in \Omega \) and for some \( n > 0 \). Prove that either \( f \) has finitely many zeros in \( \{ |z| > 2 \} \) or \( f \) is identically zero.

3. Let \( f \) be an entire function that is not a polynomial. Denote
   \[
   M(r) = \max_{|z|=r} |f(z)|.
   \]
   Show that
   \[
   \lim_{r \to \infty} \frac{M(r/2)}{M(r)} = 0.
   \]

4. Let \( f \) and \( g \) be analytic functions in the same connected complex domain \( \Omega \). Suppose that \( |f| = \Re g \) in \( \Omega \). Show that \( f \) and \( g \) are constants.

5. Consider the line in the \( z \) plane defined by the following equation:
   \[
   3\Re(z) + 4\Im(z) = 5.
   \]
   Under the inversion that sends \( z \) to \( 1/z \), this line transforms into a circle. Find the center and the radius of that circle.

6. Consider a rational function \( f(z) = q(z)/p(z) \), where \( p \) is a polynomial of degree \( n \) and \( q \) is a polynomial of degree \( n - 2 \) or less. If \( z_1, z_2, \ldots, z_n \) are distinct roots of \( p \), prove that the residues of \( f \) satisfy
   \[
   \sum_{k=1}^{n} \text{Res}(f, z_k) = 0.
   \]

7. Let \( f \) be an entire function. Prove that all the coefficients in the power series expansion of \( f \) at the origin are real if and only if \( f \) is real on the real line.

8. Find a biholomorphic map between the unit disk and the parabolic region in the \( z \) plane defined by the property that \( \Im(z) > (\Re(z))^2 \).

9. Use harmonic functions to prove the following statement: For any continuous function \( f \) on the unit circle \( \mathbb{T} = \{ |z| = 1 \} \) there exists a sequence of polynomials \( p_n(z, \bar{z}) \) of \( z \) and \( \bar{z} \) that converges to \( f \) uniformly on \( \mathbb{T} \) (The Weierstrass Approximation Theorem for the unit circle).

10. Show that there is no entire function of finite order, except the zero function, that has roots at all points \( z \) such that \( \exp(\exp(z)) = 1 \).