

2011 Power Team Rules

- (1) Each power team entry must have a cover sheet (typed). The cover sheet must contain the name of the school, the coach's name, the team name, and the names of each team member. For example, if a school has 3 power team entries, then there should be three different team names. The following is an example of an acceptable title page:

2011 Power Team Entry
XXXX School, Team 1
Coach: Ms. Wizard

Team Members:

Jane Doe

John Smith

etc.

- (2) Team participants are not allowed to consult with anyone but their team mates. Participants are not allowed to look on the web for any information regarding the power team exam, nor are they allowed to search books or other reference materials.
- (3) Team entries are expected to be neat and legible. If not, they face the possibility of being disqualified by the judges.
- (4) All hand delivered submissions are due by 9:15 am on the day of the contest! FAXed submissions will be accepted provided they are received no later than 8:30 AM. on the day of the contest. FAXed solutions should be sent to 979-862-4190, attention Mike Stecher.

Power Team Exam

Texas A&M University High School Math Contest

Nov. 12, 2011

Definition 1. Let $m \geq n \geq k \geq 1$ be natural numbers. We say that a set M of points in the plane has the property (m, n, k) if M has m points and among any n points in the set M , one can always find at least k points that can be covered by a disk of diameter 1. Note: we assume a disk always includes its boundary.

For example, in the set of points below each point is 1 unit away from its nearest neighbor, and this set has property $(4,3,2)$ but not property $(4,2,2)$.



Problem 1. Let $m \geq n \geq k \geq 2$ be natural numbers. A set M of m points with the property (m, n, k) is given in the plane. Show that the minimal number of disks of radius 1 (diameter 2) that is always sufficient to cover all the points in M is $n - k + 1$. What can you say if $k = 1$?

Definition 2. The symbol $\#(m, n, k)$ is defined to equal the minimal number of disks of diameter 1, which will cover any set with property (m, n, k) .

Note: to show that the minimal number of disks that are always sufficient is N , one needs to not only verify that N disks are good enough regardless of the position of the points, but also that for some special positions one has to use N disks.

The remaining problems in this exam are related to the following general question.

Question 1. Let $m \geq n \geq k \geq 1$. What is the minimal number of disks of diameter 1 that is always sufficient to cover the points of a set with property (m, n, k) ? That is, what is $\#(m, n, k)$?

For example

$$\begin{aligned} \#(2, 2, 1) &= 2, & \#(2, 2, 2) &= 1 \\ \#(3, n, 1) &= 3, & \#(3, 2, 2) &= 2, & \#(3, 3, 2) &= 2, & \#(3, 3, 3) &= 1 \end{aligned}$$

Note that it follows from the solution of Problem 6 that

$$(1) \quad \#(4, 3, 3) = 1.$$

For problems **2** through **4** you may assume that not only is (1) correct, but the following more general statement is also correct.

$$(2) \quad \#(m, 3, 3) = 1, \text{ for } m \geq 3.$$

Problem 2. Calculate the following numbers (be sure to carefully justify your answer in each case).

(a) $\#(4, 2, 1)$, (b) $\#(4, 2, 2)$, (c) $\#(4, 3, 1)$, (d) $\#(4, 3, 2)$.

Problem 3. Calculate the following numbers (be sure to carefully justify your answer in each case).

(a) $\#(5, 4, 2)$, (b) $\#(5, 4, 3)$, (c) $\#(5, 4, 4)$

Problem 4. Calculate the following numbers (be sure to carefully justify your answer in each case).

(a) $\#(6, 5, 2)$, (b) $\#(6, 5, 3)$, (c) $\#(6, 5, 4)$

Problem 5. Four disks of diameter 1 are placed in the plane. If every proper subcollection of the four disks have a common point, then all four disks have a common point.

Problem 6. Four points are placed in the plane. If every proper subset of these four points can be covered by a disk of diameter 1, then all four points can be covered by such a disk.