Listening to Distance

We begin with a much simplified model of reflection seismology. Suppose an explosive charge is detonated at some point on the (flat) surface of the earth. A pressure wave then spreads out from the explosive source and propagates through the subsurface. The speed of the wavefront depends on the composition of the subsurface and generally increases with the density of the material. Knowledge of the velocity of propagation can therefore give information about the nature of the material in the subsurface. In our very simple model we assume a homogeneous, horizontally stratified subsurface of constant depth d in which the velocity of propagation of the pressure wave is the constant v. Below the surface layer is hard bedrock off of which the pressure wave is (partially) reflected to eventually arrive again at the surface (see the attached figure).

Suppose a receiver, called a geophone, located X units distant from the source detects the reflected signal at a time T after the detonation of the charge. Clearly the relationship between the quantities d, v, X, and T depends on the actual path traced by the ray. The physical principle that determines the path is due to Fermat. Fermat's Principle holds that the travel time along the path must be a minimum. Since we are assuming that the velocity of propagation is constant, this amounts to saying that the total length of the path must be a minimum.

Exercises:

Problems 1-8 deal with the scenario described above. Problems 9-13 are related questions. Each problem will be graded based upon the correctness of the solution, the novelty of the approach, and the clarity of the explanation.

1. Use Fermat's Principle to show that the triangle in the figure must be isosceles and that the velocity of propagation is given by

$$v = \frac{2s}{T}$$

   where s is the distance from the source to the point of reflection.

2. Show that d, v, X, and T are related by the equation

$$T^2v^2 - 4d^2 = X^2.$$

3. A reflected seismic signal is heard at a geophone 300 meters away from the source 3 seconds after detonation. What is the slowest possible velocity of propagation, irrespective of the depth of the stratum? Explain.

4. A reflected seismic signal is received 300 meters from the source after 3 seconds. Is it possible for the signal to be received at a geophone 360 meters from the source after 4 seconds?

5. A reflected seismic signal is received 91 meters from the source after 1.1 seconds. The reflected signal is received at a second location 200 meters from the source after 2.1 seconds. Find the depth of the stratum and the velocity of propagation.

6. Suppose a reflected signal is detected 140 meters from the source after 2.1 seconds. Suppose the same signal is heard 400 meters from the source after 5 seconds. Find the depth of the stratum and the velocity of propagation in the stratum.

7. Suppose (X_1, T_1) and (X_2, T_2) are two distance-time observations satisfying $X_1 < X_2$ and $T_1 < T_2$. Find a condition on these values that ensures the existence of a unique depth d and propagation velocity v for the stratum.

8. For given values of the depth d and velocity v, the travel time T is a function of the horizontal distance X. Discuss the behavior of this travel-time function as X becomes very small or very large. Give physical interpretations of your conclusions.

9. In his travels in Italy, Johann Wolfgang von Goethe made the following diary entry for October 7, 1786, concerning the singing of Venetian gondoliers: "The singer sits on the shore of an island, on the bank of a canal or in a gondola, and signs to the top of his voice. His aim is to make his voice carry as far as possible over the still mirror of water. Far away another singer hears it. He knows the melody and the words and answers with the next verse. The first singer answers again, and so on. They keep this up night after night without ever getting tired." Suppose a listener is positioned on the line segment between two singing gondoliers, A and B, who sing according to Goethe's description (i.e. B is silent while A sings, and B begins to sing immediately upon hearing the last note from A, etc.). One second after hearing the last note from A, the listener hears the first note from B. One and a half seconds after the listener notices that B has stopped singing, he hears the first note of A's reply. How far apart are the gondoliers and where is the listener positioned? (Use 1,100 ft/sec for the speed of sound.)

10. Suppose two people are at different positions, say P and Q, in the xy-plane. Let D be the distance between P and Q. Lightning strikes at another point in the plane. Let t_P be the elapsed time between the instant the observer at P sees the lightning flash and the instant at which he hears the clap of thunder, and let t_Q stand for the corresponding time for the observer at Q. For definiteness, suppose $t_P \leq t_Q$.

    (a) Did the lightning strike nearer to P or to Q?

    (b) Let r_P be the distance from P to the lightning strike and similarly for r_Q. Is $r_Q + r_P < D$ possible?

(c) Describe the set of possible locations of the lightning strike if $r_P = r_Q$.

(d) Is $r_Q > D + r_P$ possible?

(e) Is $D < r_Q + r_P < r_P + D$ possible?

11. Two people, sitting in their houses, which are 10,000 feet apart, and talking on the telephone, hear the same clap of thunder. One person hears the thunder 4 seconds after the other person. Find the curve consisting of the possible positions of the lightning strike. The positions of the houses should be shown in your plot.

12. Take the speed of light to be essentially infinite in comparison to the speed of sound. If in the previous problem the observer nearer to the lightning strike hears the thunderclap 3 seconds after seeing the flash, estimate the possible locations of the strike.

13. Suppose a small isolated mountain has a base radius of 1 mile. An explosion took place at ground level away from the mountain, and a man standing 1 mile due east of the base of the mountain claims that he heard one loud explosion followed by a smaller explosion. He also claims that he heard the second explosion approximately 3 seconds after he heard the first explosion. Why did the man hear two explosions? Determine the curve of all possible points at which the explosion could have occurred.