AB EXAM
TEXAS A&M HIGH SCHOOL MATH CONTEST
NOVEMBER 16, 2013

This exam has 21 problems.

Directions: If units are appropriate, include them in your answer. All answers must be in reduced form.

1. The average of a and b is x. The average of b and x equals the average of a and $b+1$. Find $a-b$.

2. If $x+2y=4$, what is the perimeter of a square with side length $2x^2+8xy+8y^2$?

3. The sides of a right triangle are $m, m+n$ and $m+2n$, where both m and n are positive. Find $\dfrac{m}{n}$.

4. On his recent safari, Dr. Eb Tide noted that his group, consisting of men and elephants, had a total of 100 knees and 100 trunks. If each man packed three trunks and each elephant had the usual one, what is $\dfrac{E}{M}$, the ratio of the number E of elephants to the number M of men?

5. Let g be a function such that

$$3g(x)+2g(1-x)=9+2x$$

for all x. Find the value of $g(2)$.

6. If p and q are the roots of $x^2-6x+2=0$, find the value of $\dfrac{1}{p}+\dfrac{1}{q}$.

7. Hasse begins a city stroll at an intersection. He walks north one block and then flips a coin. If the coin shows heads, Hasse turns right, if it is tails he turns left. In all he walks a total of four blocks, flipping the coin at each corner to decide which way to turn. What is the probability that Hasse ends at the starting point?

8. Starting with the point $(4,2)$ in the coordinate plane, reflect it across the x-axis, then rotate it 180° around the origin and finally translate it vertically up by 2 units. What are the coordinates of the point's final location?

9. Let f be the function such that

$$f(x/5)=x^2+x+3\,.$$

Find the sum of all values w such that $f(5w)=2013$.

10. How many distinct permutations can be made from the letters in the word INFINITY?

11. Find all real numbers x such that $\sqrt{4-x}+\sqrt{4+x}=2x$.

12. How many integers between 1 and 2013, inclusive, are divisible by neither 3 nor 7?

13. Consider the two digit number x, which has value y when it's digits are reversed. Find x when $4x-5y=4$.

14. Given that $x+\dfrac{1}{x}=3$, find the value of $x^4+\dfrac{1}{x^4}$.

15. Consider the three-digit number $95d$, where d is the units digit. If the digits are reversed and the resulting number is subtracted from the original number, the result will consist of the same three digits in a different order. Find d.

16. Out of the numbers $2^{500}, 3^{400}, 4^{300}, 5^{200}$ which is the smallest?

17. Find the area of the region in the plane consisting of all points (x, y) that satisfy $|4x-8| \leq y \leq 12$.

18. What is the largest integer n such that $\dfrac{n^2-38}{n+1}$ is an integer?

19. We are given the following sequence:

    SUPERSOLVERSUPERSOLVERSUP...

    If the pattern continues, what letter will be in the 2013^{th} position?

20. Let $S=(2013)^0+(2013)^1+(2013)^2+\cdots+(2013)^{2013}$. Find the remainder when S is divided by 7.

21. In a random arrangement of the letters of BLUEMOUNTAINS , what is the probability that the vowels are in alphabetical order within the arrangement? Express your answer as a rational number in lowest terms.