BC EXAM
Texas A&M High School Math Contest
November 2013

Directions: If units are involved, include them in your answer.

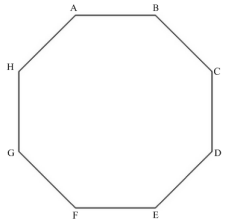
1. An aquarium has a rectangular base that measures $100cm$ by $40cm$ and has a height of $50cm$. It is filled with water to a height of $40cm$. A brick with a rectangular base that measures $40cm$ by $20cm$ and a height of $10cm$ is placed in the aquarium. By how many centimeters does the water rise?

2. How many non-congruent triangles with perimeter 7 have integer side lengths?

3. Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m. What is $x + y + m$?

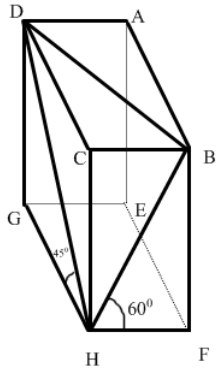
4. A two-digit integer x is to be chosen. If all choices are equally likely, what is the probability that at least one digit of x is 7?

5. A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of rectangle $ABEF$?



6. Find a positive four-digit integer which has the decimal representation $(abba)_{10}$ and it is a perfect cube.

7. In the rectangular solid shown, we have $\angle DHG = 45°$ and $\angle FHB = 60°$. What is the cosine of $\angle BHD$?



8. If positive integers x and y satisfy the equation $xy = 3(x + y) - 5$ and $x \neq y$, find $x + y$.

9. Find the value of the constant k for which the graphs $2y + x + 3 = 0$ and $3y + kx + 2 = 0$ are perpendicular.

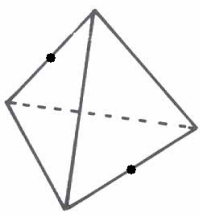
10. What is the sum of the digits in the number $10^{55} - 55$?

11. Let a, b, and c be positive integers satisfying the following system of equations:

$$\begin{cases} 7a^2 - 3b^2 + 4c^2 = 8 \\ 16a^2 - 7b^2 + 9c^2 = -3 \end{cases}$$

Find $a^2 + b^2 + c^2$.

12. An insect lives on the surface of a regular tetrahedron with edges of length 1. It wishes to travel on the surface of the tetrahedron from the midpoint of one edge to the midpoint of the opposite edge (see the picture). What is the length of the shortest such trip (Note that two edges of a tetrahedron are *opposite* if they have no common endpoint).



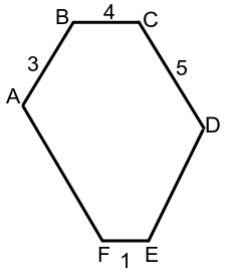
13. Find the following sum $1 + 4 + 7 + \ldots + 94 + 97 + 100$.

14. Calculate
$$\frac{8 + 222 \cdot 444 \cdot 888 + 444 \cdot 888 \cdot 1776}{2 \cdot 4 \cdot 8 + 444 \cdot 888 \cdot 1776 + 888 \cdot 1776 \cdot 3552}$$

15. Given p and q are positive integers satisfying the following equality $p! + 12 = q^2$, find $p + q$. (Recall that $p! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (p-1) \cdot p$.)

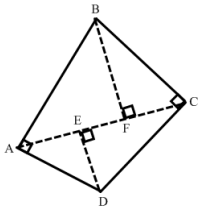
16. The sides of a triangle satisfy the following inequalities:
$$a \leq 5 \leq b \leq 6 \leq c \leq 8.$$
Find the maximum possible area of such triangle.

17. Each angle of the hexagon $ABCDEF$ is 120°. Find $AF + DE$ if $AB = 3$, $BC = 4$, $CD = 5$ and $EF = 1$.



18. Let $\spadesuit(n)$ denote the sum of the digits of the positive integer n. For example, $\spadesuit(5) = 5$ and $\spadesuit(8123) = 8 + 1 + 2 + 3 = 14$. For how many two-digit values of n is $\spadesuit(\spadesuit(n)) = 3$?

19. The quadrilateral $ABCD$ has right angles at A and at C. Points E and F are on AC with DE and BF perpendicular to AC. In addition, $AE = 3$, $DE = 5$, and $CE = 7$. What is BF?



20. Three cubes having volumes 1, 8, and 27 are glued together at their faces. What is the smallest possible surface area that the resulting polyhedron can have?