

**CD Exam**  
**Texas A&M High School Math Contest**  
**Nov. 16, 2013**

Answers should include units when appropriate.

1. Ann and Barbara were comparing their ages and found that Barbara is as old as Ann was when Barbara was as old as Ann had been when Barbara was half as old as Ann is. If the sum of their present ages is 44, what is Ann's age?

2. The side of an equilateral triangle is  $s$ . A circle is inscribed in the triangle and a square is inscribed in the circle. Find the area of the square.

3. Solve the equation

$$|x + 1| - |x| + 3|x - 1| - 2|x - 2| = x + 2.$$

4. Find the sum  $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n-1)(2n+1)} + \cdots + \frac{1}{2011 \cdot 2013}$ .

5. All even numbers from 2 to 98 inclusive, except those ending in 0, are multiplied together. What is the rightmost digit (the units digit) of the product?

6. Find the set of all real solutions of the inequality

$$|x - 1| + |x + 2| < 5.$$

7. The sequence

$$1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, \dots$$

consists of 1's separated by blocks of 2's with  $n$  2's in the  $n$ th block. Find the sum of the first 1234 terms of the sequence.

8. What is the smallest integral value of  $k$  such that

$$2x(kx - 4) - x^2 + 6 = 0$$

has no real roots?

9. Given a circle of radius 2, find the area of the region consisting of all segments of length 2 that are tangent to the circle at their midpoint.

10. Find positive integers  $A$ ,  $B$ , and  $C$ , with no common factor greater than one such that

$$A \log_{200} 5 + B \log_{200} 2 = C.$$

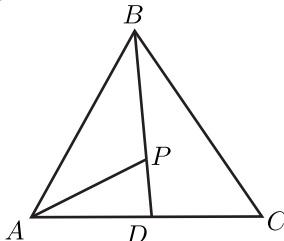
11. Let  $p$ ,  $q$ , and  $r$  be distinct roots of  $x^3 - x^2 + x - 2 = 0$ . Find  $p^3 + q^3 + r^3$ .

12. Solve the equation  $\sqrt{2 - \sqrt{2 + x}} = x$ .

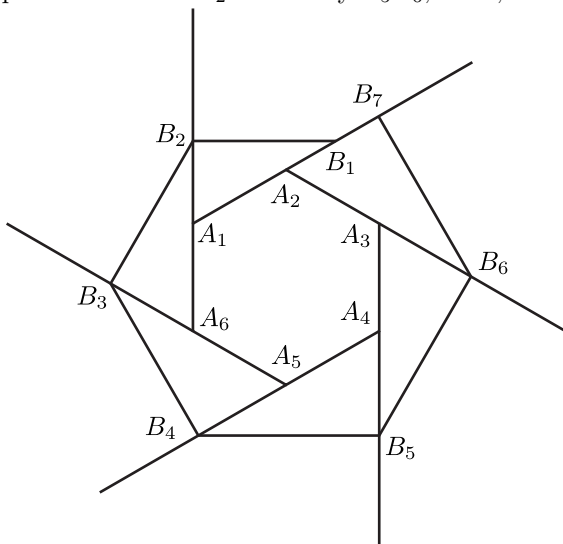
13. Solve the system

$$\begin{cases} x^2 + y = 3/4 \\ x + y^2 = 3/4 \end{cases}$$

14. Triangle  $ABC$  and point  $P$  in the same plane are given, such that  $PA = PB$ , angle  $\angle APB$  is twice angle  $\angle ACB$ , and  $\overline{AC}$  intersects  $\overline{BP}$  at point  $D$ . If  $PB = 3$  and  $PD = 2$ , find  $AD \cdot CD$ .

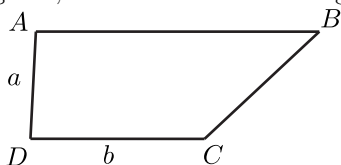


15. Points  $A_1, A_2, A_3, A_4, A_5, A_6$  are vertices of a regular hexagon inscribed in a circle of radius 1.  $B_1$  is a point on the ray  $A_1A_2$ ,  $B_2$  is the base of the perpendicular from  $B_1$  to the ray  $A_6A_1$ ,  $B_3$  is the base of the perpendicular from  $B_2$  to the ray  $A_5A_6$ , e.t.c., see the picture. If  $B_7$  coincides with  $B_1$ , then find  $A_1B_1$ .

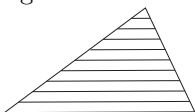


16. Find the number of ways 10 dollars can be changed into dimes and quarters, with at least one of each coin being used.

17. In the quadrilateral  $ABCD$ , segments  $AB$  and  $CD$  are parallel, the measure of angle  $D$  is twice that of angle  $B$ , and the measures of segments  $AD$  and  $CD$  are  $a$  and  $b$  respectively. Find the measure of  $AB$ .



18. Nine lines parallel to the base of a triangle divide the other sides each into 10 equal segments and the area into 10 distinct parts. If the area of the largest of these parts is 38, then what is the area of the original triangle?



19. Find all polynomials  $P(x)$  such that  $P(0) = 0$  and  $P(x^2 + 1) = (P(x))^2 + 1$ .

20. Find all positive integers  $x, y, z, t$  such that

$$\begin{cases} x + y = zt \\ z + t = xy \end{cases}$$

and  $x \leq y, z \leq t$ .

21. Find integers  $a_2, a_3, a_4, a_5, a_6, a_7$  such that  $0 \leq a_i < i$  for  $i = 2, 3, \dots, 7$ , and

$$\frac{5}{7} = \frac{a_2}{2!} + \frac{a_3}{3!} + \frac{a_4}{4!} + \frac{a_5}{5!} + \frac{a_6}{6!} + \frac{a_7}{7!}.$$

Here  $n!$  denotes the product of all integers from 1 to  $n$ .