2014 Power Team Rules

(1) Each power team entry must have a cover sheet (typed). The cover sheet must contain the name of the school, the coach's name, the team name, and the names of each team member. For example, if a school has 3 power team entries, then there should be three different team names. The following is an example of an acceptable title page:

2014 Power Team Entry
XXXX School, Team 1
Coach: Ms. Wizard

Team Members:
Jane Doe
John Smith
etc.

(2) Team participants are not allowed to consult with anyone but their team mates. Participants are not allowed to look on the web for any information regarding the power team exam, nor are they allowed to search books or other reference materials.

(3) Team entries are expected to be neat and legible. If not, they face the possibility of being disqualified by the judges.

(4) All hand delivered submissions are due by 9:15 am on the day of the contest. FAXed submissions will be accepted provided they are received no later than 8:30 AM. on the day of the contest. FAXed solutions should be sent to 979-862-4190, attention Mike Stecher.

2014 POWER TEAM EXAM
TEXAS A&M HIGH SCHOOL MATH CONTEST
8 NOVEMBER 2014

Problem 1. For which values of the real parameter a ($a \neq -1$) are the solutions of the quadratic equation

$$(a+1)x^2 - 3ax + 4a = 0$$

real and greater or equal to -1?

Problem 2. Let a be a real-valued parameter for which the solutions x_1 and x_2 of the quadratic equation

$$x^2 + ax + 1 = 0$$

are real.

(a) Find coeficients b and c in terms of a such that $y_1 = x_1(1 - x_1)$ and $y_2 = x_2(1 - x_2)$ are solutions of $y^2 + by + c = 0$.

(b) For what values of the parameter a are both solutions y_1 and y_2 between -2 and 1?

Problem 3. For which values of the real-valued parameter a does the set of solutions of the equation

$$(x-2)^2 = |x - a|$$

consist of three distinct real numbers?

Problem 4. For a nonzero real number a, let x_1 and x_2 be the solutions of the quadratic equation

$$x^2 + ax - \frac{1}{2a^2} = 0.$$

What is the smallest possible value of $x_1^4 + x_2^4$?

Problem 5. Let a be a real-valued parameter. Find all real solutions of the equation

$$\sqrt{x^2 - 2a} + \sqrt{4x^2 - a - 2} = x.$$

Problem 6. For each real number a, let $f(a)$ be the larger of the two solutions of the quadratic equation

$$(a^2+1)x^2 + 10ax - 6(9a^2+1) = 0.$$

What are the largest and the smallest possible values of $f(a)$?

Problem 7. Find the smallest possible value of the real parameter a for which the equation

$$|x+1| + |x+2| + |x+3| + \cdots + |x+2014| = a$$

has a solution and solve the equation for that value of a.

Problem 8. For which values of the parameter a does the equation

$$|||||x| - 1| - 2| - 3| - a| = 2014$$

have an odd number of solutions?

Problem 9. Determine as large a value of a as you can so that

$$\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \frac{1}{\sqrt{9}+\sqrt{11}} + \cdots + \frac{1}{\sqrt{9997}+\sqrt{9999}} > a.$$

(Note: it is not required to find the best possible value of a, just as large as you can find – the higher the value of a which you prove works, the higher the mark in the grading).

Problem 10. Determine all values of the positive integer n for which the system of equations

$$\begin{aligned} x_1 + x_2 + \cdots + x_n &= 9 \\ \frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n} &= 1 \end{aligned}$$

has positive real solutions and determine all such solutions.

Problem 11. (a) Determine the largest value of the parameter a such that the inequality

$$x^2 + y^2 + z^2 \geq a(xy + yz)$$

is valid for all real numbers x, y, and z.

(b) Determine the largest value of the parameter b such that the inequality

$$x^2 + y^2 + z^2 + w^2 \geq b(xy + yz + zw)$$

is valid for all real numbers x, y, z, and w.