Answers should include units when appropriate.

1. Two ferry boats ply back and forth across a river with constant speeds, turning at the banks without loss of time. They leave opposite shores at the same instant, meet for the first time 700 feet from one shore, continue on their way to the banks, return and meet for the second time 400 feet from the opposite shore. Determine the width of the river.

2. Three men, A, B, and C, working together, do a job in 6 hours less time than A alone, in 1 hour less time than B alone, and in one-half the time needed by C when working alone. What is the time needed by A and B, working together to do the job.

3. Find the smallest integer \( n \) such that \( \sqrt{n} - \sqrt{n-1} < 0.01 \).

4. Find all possible values of the expression

\[
\frac{|x+y|}{|x|+|y|} + \frac{|y+z|}{|y|+|z|} + \frac{|z+x|}{|z|+|x|},
\]

where \( x, y, z \) are arbitrary non-zero real numbers.

5. Solve the system

\[
\begin{align*}
x &= x^2 + y^2 \\
y &= \frac{2xy}{5}
\end{align*}
\]

6. If \( \sin x + \cos x = 1/5 \) and \( 0 \leq x < \pi \), find \( \tan x \).

7. Let \( AB \) be a diameter of a circle. Tangents \( AD \) and \( BC \) are drawn so that \( AC \) and \( BD \) intersect in a point on the circle. If \( AD = a \) and \( BC = b \), find the diameter of the circle.

8. Let \( a, b \) be positive real numbers. Find the values of \( m \) for which the equation

\[
|x-a| + |x-b| + |x+a| + |x+b| = m(a+b)
\]

has at least one real solution.

9. If \( \cos 2\alpha = m \), find \( \sin^6 \alpha + \cos^6 \alpha \).

10. Find the remainder obtained by dividing \( x^{2015} \) by \( x^2 - 3x + 2 \).

11. Solve the system

\[
\begin{align*}
\log_y x + \log_x y &= 5/2 \\
x y &= 27
\end{align*}
\]

12. Solve the equation

\[
(\sqrt{0.5} + \sqrt{4})^x = 13.5.
\]

13. Find all pairs \((x, y)\) of real numbers such that \(x^2 + 2x \sin(xy) + 1 = 0\).
14. A circle is tangent to the coordinate axes and to the hypotenuse of the $30^\circ - 60^\circ - 90^\circ$ triangle $ABC$ as shown, where $AB = 1$. Find the radius of the circle.

15. In a quadrilateral $ABCD$, it is given that $\angle A = 120^\circ$, angles $B$ and $D$ are right angles, $AB = 13$, and $AD = 46$. Find $AC$.

16. Find all values of $c$ such that the polynomials $cx^3 - x^2 - x - (c+1)$ and $cx^2 - x - (c+1)$ have a common root.

17. Find all $x$ such that $0 \leq x < 2\pi$ and $\sin 3x = 2 \sin x$.

18. In a triangle $ABC$, $AD$ and $AE$ trisect $\angle BAC$. The lengths of $BD$, $DE$, and $EC$ are 2, 3, and 6, respectively. Find the length of $AB$.

19. In the quadrilateral $ABCD$, segments $AB$ and $CD$ are parallel, the measure of angle $D$ is twice that of angle $B$, and the measures of segments $AD$ and $CD$ are $a$ and $b$ respectively. Find the measure of $AB$.

20. Suppose that the function $f(n)$ satisfies $f(x) + f(y) = f(x + y) - xy - 1$ for every pair $x, y$ of real numbers. If $f(1) = 1$, find all integers $n$ such that $f(n) = n$.

21. If $P(x)$ denotes a polynomial of degree $n$ such that $P(k) = \frac{k}{k+1}$ for $k = 0, 1, 2, \ldots, n$, determine $P(n+1)$. 