2017 AB Exam

Texas A&M High School Math Contest
October 21, 2017

1. Solve for x : $\sqrt{x-2} = 4 - x$.

2. How many seven digit positive integers are palindromes? (A palindrome reads the same forwards and backwards, for example 1234321 is a palindrome.)

3. When the positive integer n is divided by 39 the remainder is 10. What is the remainder when $5n$ is divided by 39?

4. If $f(x) = 3x + 4$ find a function g such that $f(g(x)) = 4x - 1$.

5. Suppose f is a linear function with $f(0) = 6$ and $f(2) = 3$. Find $f(1)$.

6. Find all values of m such that the line $y = mx + 3$ intersects the curve $y = x^2 + 2x + 7$ at exactly one point.

7. Find the minimum possible value of $2x^2 + 2xy + 4y + 5y^2 - x$.

8. For which base $b > 10$ is 103_b divided by 4_b equal to 29_b?

9. Find all integer pairs (a, b) such that $ab + a - 3b = 5$.

10. Find a monic polynomial of degree 4 (coefficient of x^4 is 1) with integer coefficients having $\sqrt{2} - \sqrt{5}$ as a root.

11. Consider the infinite sequence of ordered pairs of integers:
$(1, 2017), (2, 2018), (3, 2019), (4, 2020), ...$
How many ordered pairs (a, b) are in this sequence where a divides b?

12. If the sum of the first n terms of an infinite sequence is $n^2 + 2n$, what is the 2017th term of the sequence?

13. Find the area of the region in the 1st quadrant bounded by the graphs of $y = 0$, $x = 0$, and $x^2 - x + y^2 - y = 0$.

14. A 1, 2, $\sqrt{3}$ right triangle has vertices A, B, C with B being the vertex at the right angle. A point P is randomly chosen within the triangle. What is the probability that P is within a distance of 1 from either A or C?

15. If the roots of $x^3 + ax^2 + bx + c$ are three consecutive positive integers, find the value of $\frac{a^2}{b+1}$.

16. Let A, B, C be randomly chosen and not necessarily different integers between 0 and 4 inclusive. What is the probability that $A + BC$ gives the same value as $(A + B) \cdot C$?

17. Mike the chemist has a 200gm mixture that contains 60% metal A and 40% metal B. How many grams of metal A must he add to the mixture to have one that contains 80% metal A?

18. Five different items in a store have lost their price tags. The five price tags are mixed up! Daisy the store clerk randomly attaches the tags to the five items. What is the probability that she attaches exactly two tags to correct items?

19. Find all polynomials P of degree 2 having real roots such that

$$P(x^2) = P(x)P(-x).$$

20. Find all ordered pairs of numbers (x, y) that satisfy both

$$x(x + y) = 9 \text{ and } y(y + x) = 16.$$

21. How many positive integers less than or equal to 2016 are <u>not</u> relatively prime to 2016?
(Note: $2016 = 2^5 \cdot 3^2 \cdot 7$)