1. Let $A$ be the area of triangle $ABC$. The area of the shaded region is

$$
\frac{1}{4}A + \frac{1}{4}A + \frac{1}{4}A + \frac{1}{16}A = \frac{13}{16}A ,
$$

so the fraction of the total area that is shaded is $13/16$.

2. Now $2^{100} < 5^{50} < 3^{75}$, since

$$
2^{100} = 4^{50} < 5^{50} < 3^{75} = 27^{25}
$$

$5^{50} = 25^{25}$ \(\Rightarrow\) $5^{50} < 3^{75}$

3. From the table, Boston and Toronto tied. Since Montreal defeated Boston 3-0, this accounts for all 3 goals scored against Boston. So in the Toronto-Boston game, Boston scored 0 goals. Hence there was a 0-0 tie.

4. Let $P$ be the set of applicants taking physics and let $C$ be the set of applicants taking chemistry. Then

$$
|P \cup C| = 100 - 10 = 90
$$

$$
|P \cup C| = |P| + |C| - |P \cap C|
$$

$$
90 = 75 + 83 - |P \cap C|
$$

$$
|P \cap C| = 158 - 90 = 68 .
$$

5. Since the sum is even, one of the primes must be 2. So we seek primes $p$ and $q$ with $p + q = 38$. By trial and error the primes are 7 and 31. So $(7)(31)=217$.

6. Let $x = \#$ of cards Paul had. Then

$$
\frac{1}{2}x + \frac{1}{6}x + 12 = x
$$

$$
x - \frac{1}{2}x - \frac{1}{6}x = 12
$$

$$
\frac{6}{6}x = 12
$$

$$
\frac{1}{3}x = 12
$$

$$
x = 36 .
$$
7. \(8 \cdot 10^{18} + 1^{18} = 8 \cdot (9 + 1)^{18} + 1\). Now 9 divides every term in the expansion \((9 + 1)^{18}\), except the last one, 1, giving a remainder of 1. Since \(8 \cdot 1 + 1 = 9\), there is a final remainder of 0.

8.

\[
\begin{array}{c}
\text{A} & \text{B} \\
\text{D} & \text{C}
\end{array}
\]

\[
\begin{array}{c}
16 & 8 \\
8 & 16
\end{array}
\]

\(\text{AREA}=64\)

\[
\begin{array}{c}
\text{A} & \text{B} \\
\text{D} & \text{C}
\end{array}
\]

\[
\begin{array}{c}
8 & 8 \\
16 & 8
\end{array}
\]

\(\text{AREA}=32\)

\[
\begin{array}{c}
\text{A} & \text{B} \\
\text{D} & \text{C}
\end{array}
\]

\[
\begin{array}{c}
8 & 8 \\
16 & 8
\end{array}
\]

\(\text{AREA}=24\)

9. Unfold the cylinder to obtain a rectangle.

\[
x^2 = 5^2 + 12^2 = 25 + 144 = 169
\]

\(x = 13\).

10. Now

\[
\frac{3 \quad 4 \quad 5 \quad 6 \quad a}{2 \quad 3 \quad 4 \quad 5 \quad b} = \frac{a}{2}.
\]

We need \(\frac{a}{2} = 9\) or \(a = 18\). Then \(b = a - 1 = 17\). So \(a + b = 35\).

11. \((n - 2)(n - 1)(n)(n + 1)(n + 2)\) is the product of 5 consecutive integers. At least two of the 5 must be even and differ by 2. So 8 is a divisor. At least one of the 5 is divisible by 3, so 3 is a divisor. At least one is divisible by 5, so 5 is a divisor. No other prime must divide this product. So \((8)(3)(5) = 120\) is the largest integer that must divide \((n - 2)(n - 1)(n)(n + 1)(n + 2)\) for all \(n\).

12.

\[352\]
\[532\]
\[752\]
\[572\]
\[372\]
\[732\]
\[3312\]
and \(3312/6 = 552\).

13. The basketball player must make the first shot and miss the second shot. The probability of such an occurrence is \((3/4)(1/4) = 3/16\).

14. \(2Q\) ends in the digit 4 so \(Q \in \{2, 7\}\). If \(Q=2\), then

\[
\begin{array}{c}
22 \\
\times \quad S2 \\
\hline
44 \\
\hline
xxx \\
\hline
1404
\end{array}
\]

and \(2S\) ends in 6, so \(S \in \{3, 8\}\). Neither choice for \(S\) gives 1404 as a product. So \(Q=7\) and then

\[
\begin{array}{c}
27 \\
\times \quad S2 \\
\hline
54 \\
\hline
xxx \\
\hline
1404
\end{array}
\]

and we conclude that \(S=5\). Thus \(Q+S=12\).

15.

\[
\begin{align*}
3 \\
3 + 4 \\
3 + (2)(4) \\
3 + (3)(4) \\
\vdots \\
3 + (21)(4)
\end{align*}
\]

\[
\frac{3(22)+4(1+2+\cdots+21)}{6} = 66 + 4 \left( \frac{(21)(22)}{2} \right) = 990
\]

Hence, \(15k = 990\), so \(k = 66\).

16. Let \(r\) be the radius of the smaller circle. The area of the shaded region is

\[
\frac{\pi(2r)^2}{6} - \frac{\pi r^2}{6} = \frac{9\pi}{8} \implies \frac{1}{2}\pi r^2 = \frac{9\pi}{8} \implies r^2 = \frac{9}{4} \implies r = \frac{3}{2}.
\]

The area of the smaller circle is \(\frac{9}{4}\pi\).
17. The area is \((1)(6) + ((1/2)(3))(6)\) = 15.