1. \((\frac{14}{3 + \sqrt{2}})\left(\frac{3 - \sqrt{2}}{3 - \sqrt{2}}\right) = \frac{42 - 14\sqrt{2}}{9 - 2} = \frac{7(6 - 2\sqrt{2})}{7} = 6 - 2\sqrt{2}\)

2. \(\frac{x + 2a - 3}{x + a} - \frac{x + 6}{2x} = \left(\frac{x + 2a - 3}{x + a}\right)\left(\frac{2x}{2x}\right) - \left(\frac{x + 6}{x + a}\right)\left(\frac{x + a}{x + a}\right) = \left(x + 2a - 3\right)\left(2x\right) - \left(x + 6\right)\left(x + a\right) = \frac{x + 2a - 3\left(2x\right) - \left(x + 6\right)(x + a)}{(x + a)(2x)}\)

3. \(\frac{6x^2 + 11xy - 10y^2}{3x^2 + 10xy - 8y^2} = \frac{(3x - 2y)(2x + 5y)}{(3x - 2y)(x + 4y)} = \frac{2x + 5y}{x + 4y}\)

4. \(\frac{(x - 4y^{2/5})^{3/4}}{x^{12/3}y^{-5/6}} = \frac{x^{12/3}y^{-6/20}}{x^{12/3}y^{-5/6}} = x^{3-2/3}y^{-6/20+5/6} = x^{7/3}y^{12/60} = x^{7/3}y^{2/15}\)

5. \(5(x - 7) - 13(x - 7) - 6 = 0\)
\((x - 7)(5 - 13) - 6 = 0\)
\(-8x + 56 - 6 = 0\)
\(-8x + 50 = 0\)
\(-8x = -50\)
\(x = 50/8\)

6. Multiple \(-2x + 4y = 12\) by 3 and \(3x - 5y = -3\) by 2. This gives
\(-6x + 12y = 36\)
\(6x - 10y = -6\).

Add these two equations and we get \(2y = 30 \implies y = 15\). Substituting \(y = 15\) into \(3x - 5y = -3\) gives \(x = 24\). So the point that satisfies both equations is \((24, 15)\) and the value of \(x + y\) is 24 + 15 = 39.

7. Let \(x\) be the amount invested at \(\frac{51}{2}\%\) and \(y\) be the amount invested at \(\frac{33}{4}\%\). The resulting system of equations is
\(x + y = 10,000\)
\(0.055x + 0.0675y = 650\).

Solve the system of equations to find \(x\) and \(y\).
\(x = 10,000 - y \implies 0.055(10,000 - y) + 0.0675y = 650 \implies 550 - 0.055y + 0.0675y = 650 \implies 0.0125y = 100 \implies y = 8000\). Since \(x + y = 10,000\) we know \(x = 2000\) and thus \$6,000 more is invested in \(\frac{33}{4}\%\).

8. Since \(2ax + 3by = 7c \implies 3by = 7c - 2ax \implies y = \frac{7c - 2ax}{3b} = \frac{-2a}{3b} x + \frac{7c}{3b}\). If \(x\) decreases by 10 then we know \(y = \frac{-2a}{3b} (x - 10) + \frac{7c}{3b} = \frac{-2ax}{3b} + \frac{20a}{3b} + \frac{7c}{3b}\). So \(y\) will increase by \(\frac{20a}{3b}\).

9.\(\frac{4k - 6 - 16}{2k + 3 + 2} = 0 \implies \frac{4k - 22}{2k + 5} = 0 \implies 4k - 22 = 0 \implies k = 22/4 = 5.5\).
10. (a) \( V = (10 - 2x)(6 - 2x)x = 4x^3 - 32x^2 + 60x \)
(b) \( S_A = 2x(6-2x) + 2x(10-2x) + (6-2x)(10-2x) = 12x - 4x^2 + 20x - 4x^2 + 60 - 32x + 4x^2 = -4x^2 + 60 \)
11. \( \frac{5x + 2}{x - 10} \geq 3 \implies \frac{5x + 2}{x - 10} - 3 \geq 0 \implies \frac{5x + 2 - 3(x - 10)}{x - 10} \geq 0 \implies \frac{5x + 2 - 3x + 30}{x - 10} \geq 0 \implies \frac{2x + 32}{x - 10} \geq 0. \) The expression \( \frac{2x + 32}{x - 10} \) will be greater than or equal to zero on \((-\infty, -16] \cup (10, \infty)\).
12. To find the domain of \( f(x) = \frac{\sqrt{x^2 - 3x - 4}}{6x^2 - 54} \) we know \( x^2 - 3x - 4 \geq 0 \) and \( 6x^2 - 54 \neq 0. \) \( x^2 - 3x - 4 \geq 0 \) for \((-\infty, -1] \cup [4, \infty)\) and \( 6x^2 - 54 \neq 0 \) for \((-\infty, -3) \cup (-3, 3) \cup (3, \infty)\). So the domain of \( f(x) \) is \((-\infty, 3) \cup [4, \infty)\).
13. For the function \( \frac{2x^2 + 13}{(x + 1)(x - 1)} \) on \( x < 0 \) the domain is \((-\infty, -1) \cup (-1, 0)\). For the function \( \frac{5x - 26}{x + 2} \) on \( x \geq 0 \) the domain is \([0, \infty)\). So, for the function \( f(x) \) the domain is \((-\infty, -1) \cup (-1, \infty)\).
14. \( f(x) = \frac{6x^2 - 7x - 5}{4x^2 - 12x - 7} = \frac{(2x + 1)(3x - 5)}{(2x + 1)(2x - 7)} = \frac{3x - 5}{2x - 7}, \) so the \( x \)-intercept is where \( 3x - 5 = 0 \implies x = 5/3. \)
15. \( f(x) = \frac{6x^2 - 7x - 5}{4x^2 - 12x - 7} = \frac{(2x + 1)(3x - 5)}{(2x + 1)(2x - 7)} = \frac{3x - 5}{2x - 7}, \) so the vertical asymptote is where \( 2x - 7 = 0 \implies x = 7/2. \) The horizontal asymptote is at \( y = \frac{6}{4} = \frac{3}{2}. \)
16. To find the \( x \)-intercepts solve \( f(x) = 0 \implies x(x + 3)(x - 3) = 0 \implies x = 0, \pm 3. \) To find the \( y \)-intercepts, substitute \( x = 0 \implies 0^3 - 9(0) = 0. \) Hence the \( y \)-intercept is \( y = 0. \)
17. (a) We must have \(-x^2 - 4x + 5 \geq 0 \) for \( f(x) \) to be defined. Thus \((-x + 1)(x + 5) \geq 0 \) which occurs on \([-5, 1]\). Thus the domain of \( f(x) \) is \([-5, 1]\).
(b) We must have \( 4t - 3 > 0 \) for \( g(t) \) to be defined. Thus, the domain for \( g(t) \) is \( \left(\frac{3}{4}, \infty \right) \).
(c) We must have \( x^3 + 3x^2 - x - 3 \neq 0 \) for \( h(x) \) to be defined. \( x^3 + 3x^2 - x - 3 \neq 0 \implies x^2(x + 3) - (x + 3) \neq 0 \implies (x + 3)(x^2 - 1) \neq 0 \implies (x + 3)(x - 1)(x + 1) \neq 0 \implies x \neq \pm 1 \) and \( x \neq -3. \) The domain of \( h(x) \) is \((-\infty, -3) \cup (-3, -1) \cup (-1, 1) \cup (1, \infty)\).
18. \( \left(\frac{2}{\sqrt[4]{x}}\right) \left(\sqrt[4]{4x}\right) = \left(2x^{-5/2}\right) \left(4x\right)^{1/3}\)
\( = \left(2x^{-5/2}\right) \left(4^{1/3} \cdot x^{1/3}\right) = \left(2x^{-5/2}\right) \left(2^{1/3} \cdot x^{1/3}\right) = \left(2x^{-5/2}\right) \left(2^{2/3} \cdot x^{1/3}\right) = \frac{2^{5/3}}{x^{13/6}} \)
19. The transformed graph is \( g(x) = \frac{1}{2}(x - 4)^2 + 10. \)
20. The domain of \( \log(x + 2) \) is \((-2, \infty)\). The domain of \( \log(x - 1) \) is \((1, \infty)\). So, the domain of \( \log(x + 2) + \log(x - 1) \) is \((1, \infty)\). Now, use properties of logarithms to solve the equation.
\( \log(x + 2) + \log(x - 1) = 1 \implies \log\left[\left(x + 2\right)(x - 1)\right] = 1 \implies 10^1 = (x + 2)(x - 1) \implies 10 = x^2 + x - 2 \implies 0 = x^2 + x - 12 \implies 0 = (x + 4)(x - 3) \implies x = -4, 3. \) Since \( x = -4 \) is not in the domain, \( x = 3 \) is the only solution.
21. \( 3x^2(4x^2 + 1)^8 + 64x^4(4x^2 + 1)^7 = x^2(4x^2 + 1)^7 \left(3(4x^2 + 1) + 64x^2\right) = x^2(4x^2 + 1)^7(12x^2 + 3 + 64x^2) = x^2(4x^2 + 1)^7(76x^2 + 3) \)
22. Refer to the figure below. We know that \( \tan(41°) = \frac{18}{x}. \) So, \( x = \frac{18}{\tan 41°} \) or \( x = 18 \cot(41°). \) So, the person must be \( 18 \cot(41°) \) feet from the base of the pole.

\[ \text{\[41°\] } \quad \text{18 feet} \]

23. \( f \circ g = \frac{\frac{2}{x} + 1}{\frac{2}{x} + x} = \frac{\frac{2}{x} + x}{\frac{2}{x} + x} = \frac{2}{x} \cdot \frac{x}{2 + x} = \frac{2}{2 + x} \)

24. \[
\frac{8}{x + 1} - \left( \frac{y}{z + 2} \div \frac{y - 4}{w} \right) = \frac{8}{x + 1} - \left( \frac{y}{z + 2} \cdot \frac{w}{y - 4} \right) = \frac{8}{x + 1} - \left( \frac{yw}{(z + 2)(y - 4)} \right) = \frac{8(z + 2)(y - 4) - yw(x + 1)}{(x + 1)(z + 2)(y - 4)} = \frac{8zy - 32z + 16y - 64 - ywx - yw}{(x + 1)(z + 2)(y - 4)}
\]

25. First factor the equation: \( e^{2x} - 2e^x - 3 = (e^x)^2 - 2e^x - 3 = (e^x - 3)(e^x + 1). \) Solving \((e^x - 3)(e^x + 1) = 0 \Rightarrow e^x - 3 = 0 \text{ or } e^x + 1 = 0 \Rightarrow e^x = 3 \text{ or } e^x = -1, \) but \( e^x \) will never be negative, so the only solution is \( e^x = 3 \Rightarrow x = \ln(3) \).

26. Using point slope equation we get \( y - 1 = 7(x - 5) \Rightarrow y = 7x - 35 + 1 \Rightarrow y = 7x - 34. \) Use this to find \( y \) when \( x = -4 \) gives \( y = 7(-4) - 34 \Rightarrow y = -62. \)

27. \[
\frac{f(2 + h) - f(2)}{h} = \frac{\sqrt{2 + h} + 4 - \sqrt{2 + 4}}{h} = \frac{\sqrt{6 + h} - \sqrt{6}}{h} \cdot \frac{\sqrt{6 + h} + \sqrt{6}}{\sqrt{6 + h} + \sqrt{6}} = \frac{6 + h - 6}{h(\sqrt{6 + h} + \sqrt{6})} = \frac{6 + h - 6}{h(\sqrt{6 + h} + \sqrt{6})}
\]

28. \[
\frac{(x^2y^4)^5(x^3y)^{-3}}{xy} = \frac{x^{10}y^{20}x^{-9}y^{-3}}{xy} = \frac{xy^{17}}{xy} = y^{16}
\]

29. \[
\sqrt[3]{(a^3b)^5(64a^4b^2)} = \sqrt[3]{64a^7b^5} = 4a^2b(\sqrt[3]{a})
\]

30. \[
\frac{x^2}{x^2 - x - 2} - \frac{4}{x^2 + x - 6} + \frac{x}{x^2 + 4x + 3} = \frac{x^2}{(x - 2)(x + 1)} - \frac{4}{(x + 3)(x - 2)} + \frac{x}{(x + 1)(x + 3)}
\]

31. Factor and simplify: \( f(x) = \frac{3x^2 - 14x - 5}{4x^2 - 17x - 15} = \frac{(3x + 1)(x - 5)}{(4x + 3)(x - 5)} = \frac{3x + 1}{4x + 3}. \) This yields a zero of \( x = -\frac{1}{3}, \) a vertical asymptote of \( x = -\frac{3}{4}, \) and a horizontal asymptote \( y = \frac{3}{4}. \)

32. Using the identity \( \cos^2 \theta + \sin^2 \theta = 1, \) we find that \( \cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{1}{49} = \frac{48}{49}. \) This gives us \( \cos \theta = \pm \frac{\sqrt{48}}{49} = \pm \frac{4\sqrt{3}}{7}. \) Since \( \theta \) is in quadrant II, \( \cos \theta < 0, \) hence \( \cos \theta = -\frac{4\sqrt{3}}{7}. \)

33. We know \( \ln(ab) = \ln a + \ln b; \ln \left( \frac{a}{b} \right) = \ln a - \ln b, \) and \( \ln a^b = b \ln a. \) Using these properties, we obtain \( \ln \left( \frac{\sqrt{3}x^5}{(z + 1)^2} \right) = \ln (\sqrt{3}x^5) - \ln (z + 1)^4 = \ln (x^{1/2}y^5) - 4 \ln (z + 1) = \ln x^{1/2} + \ln y^5 - 4 \ln (z + 1) = \frac{1}{2} \ln x + 5 \ln y - 4 \ln (z + 1). \)
34. \( \sec \frac{2\pi}{3} - \tan \frac{\pi}{6} = -2 - \frac{1}{\sqrt{3}} \)

35. We begin with a rectangle of area 20 in\(^2\). If we increase the length by 8%, then our new length is 5.4 inches. This gives a new area of 21.6 in\(^2\). This gives a total change in area of 1.6 in\(^2\).

36. \( f(2) = (2)^3 + 1 = 9 \) and \( f(-3) = 2(-3)^2 - 3 = 18 - 3 = 15 \), so \( f(2) - f(-3) = 9 - 15 = -6 \).

37. Use the identity \( \cos^2 \theta = 1 - \sin^2 \theta \) to get \( \cos^2(\theta) = \frac{1 - \sin^2 \theta}{1 + \sin \theta} = \frac{(1 + \sin \theta)(1 - \sin \theta)}{1 + \sin \theta} = 1 - \sin \theta \).

38. \( \log_4 \left( \frac{1}{\sqrt{16}} \right) = \log_4 \left( \frac{1}{4^{1/2}} \right) = \log_4 \left( \frac{1}{4^{2/3}} \right) = \log_4 4^{-2/3} = -2/3 \)

39. \( \frac{1}{a} - \frac{b}{n} + a = \frac{(1 - b)(ab^3)}{(1/n + a)(ab^3)} = \frac{b^3 - ab^4}{a + a^2b^3} \)

40. Use the exponential growth model \( y(t) = y_0e^{kt} \), where \( y(t) \) is the size of the population at time \( t \) and \( y_0 \) is the initial size of the population. We know that \( y_0 = 1200 \), hence \( y(t) = 1200e^{kt} \). Since the population doubles every day, we know \( 2400 = 1200e^{k \cdot 24} \implies 2 = e^{24k} \implies \ln 2 = 24k \implies k = \frac{1}{24} \ln 2 = \ln 2^{1/24} \). Thus, \( y(t) = 1200e^{t \ln 2^{1/24}} = 1200e^{\ln 2^{1/24}} = 1200 \cdot 2^{t/24} \) (note \( t \) is in hours). Now solve for \( t \) when \( y(t) = 10000 \implies 10000 = 1200 \cdot 2^{t/24} \implies \frac{10000}{1200} = 2^{t/24} \implies \frac{25}{3} = 2^{t/24} \implies \log_2 \left( \frac{25}{3} \right) = \frac{t}{24} \implies t = 24 \log_2 \left( \frac{25}{3} \right) \). So, it will take \( 24 \log_2 \left( \frac{25}{3} \right) \) hours for the culture to reach 10,000 bacteria.