Martingale polynomials and reverse martingales

Michael Anshelevich

For a (Lévy) random process $\{X_t : t \in \mathbb{R}\}$, a *martingale polynomial* is a polynomial $P(x,t)$ such that for any $s < t$, the conditional expectation (projection)

$$\mathbb{E}\left[P(X_t,t)|X_s\right] = P(X_s,s).$$

For example, obviously $P(x,t) = x$ will work since X_t itself is a martingale, and it is well-known that for the Brownian motion, $X_t^2 - t$ is a martingale. Are there other martingale polynomials, and how do we find them? Similarly, are there any *reverse* martingale polynomials, such that, still for $s < t$,

$$\mathbb{E}\left[P(X_s,s)|X_t\right] = P(X_t,t)?$$

The goal of the talk is to explain these notions and results, and the *real* goal of the talk is to present some very elementary proofs for them. Everything in the abstract will be explained, and I think (although I may be wrong) that no background beyond linear algebra would be necessary.