Optimal Harvesting Models for Fishery Populations

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Outline

Motivation

Fish worldwide are at varying population levels

- Healthy
- Declining
- Endangered

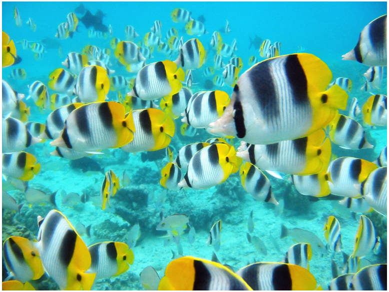
Dilemma:

- Economy demands fish for food
- Ecologically important to keep populations intact

What is fishery management?

- The study of fish populations under harvesting strategies

Research



We have been investigating:

1. Keeping a population stable
2. Harvesting under different rates (some time dependent)

Problem:

- How can we maximize yield without endangering population sustainability?

Allee effect

Definition

In population dynamics an Allee effect occurs when there is a positive correlation between population density $n(t)$ and population growth rate $\frac{dn}{dt}$.

- When the population is small there is a penalty on reproduction
- Hard for individuals to find mates
- Leads to smaller growth rate

Single stage models

We will consider three models:

1. Simple logsitic model
   $\frac{dn}{dt} = n(1-n) - fn$
2. Skewed logistic model
   $\frac{dn}{dt} = n^2(1-n) - fn$
3. Allee effect model
   $\frac{dn}{dt} = n(n-a)(1-n) - fn$

where n is fish population, f is harvest rate, and $0 < a < 1$ is a constant (Allee effect).

For each model we found the equilibrium solution. We have also determined the harvest rate that maximizes yield.

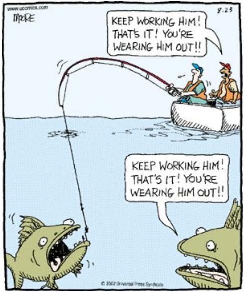
Harvesting strategies

We will consider two harvesting strategies:

1. Constant harvesting
2. Time dependent harvesting
   - f is a function of time

A constant harvesting strategy

Harvest at a rate $0 < f < 1$ each time unit.



A time dependent harvesting strategy

In theory we could harvest at any function of t

- Impractical to implement

Simplified strategy:

$$f(t) = \begin{cases} f_1 & : 0 < t < \frac{T}{2} \\ f_2 & : \frac{T}{2} < t \leq T \end{cases}$$

so we harvest for time T at two constant rates.

Yield

How do we calculate yield from the harvesting function?

$$Y := \int_0^T fn(t)dt$$

Analysis of models

Classical approach:

- Find equilibrium solution
- Determine yield

Transient approach:

- Fix parameters (initial conditions, time)
- Find harvest rate that maximizes yield

Equilibrium

Definition

An equilibrium is stable if the system returns to equilibrium after small disturbances.

Otherwise, if the system moves away from equilibrium after small disturbances then the equilibrum is unstable.

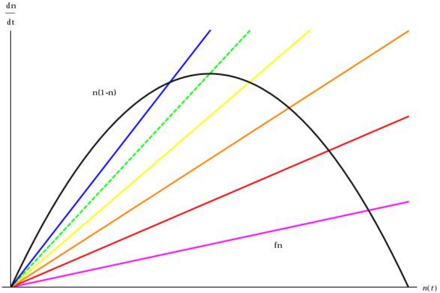
Let n^*, f^*, and Y^* be population density, harvest rate, and yield at equilibrium.

Simple logistic model - equilibrium

$$\frac{dn}{dt} = n(1-n) - fn$$

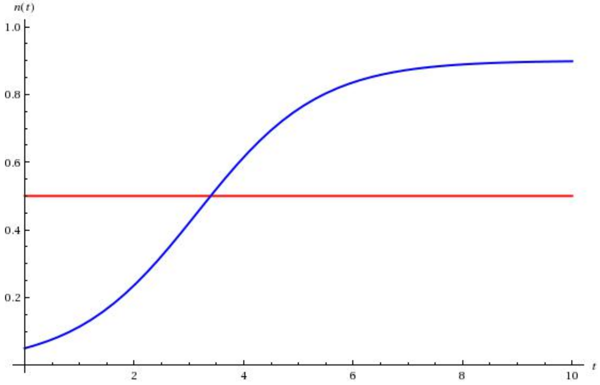
Solution:

- $n^* = \frac{1}{2}$
- $f^* = \frac{1}{2}$
- $Y^* = \frac{1}{4}$

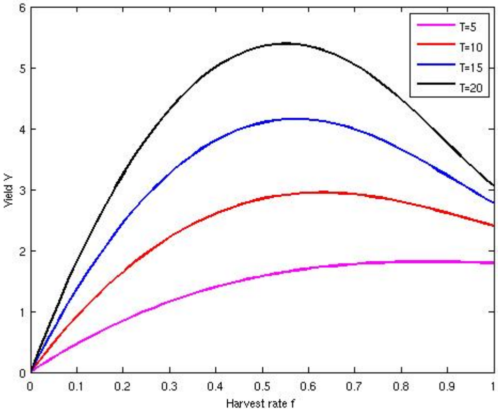


Simple logistic solution

Equilibrium population denisity is half of the carrying capacity



Simple logistic - transient (time)



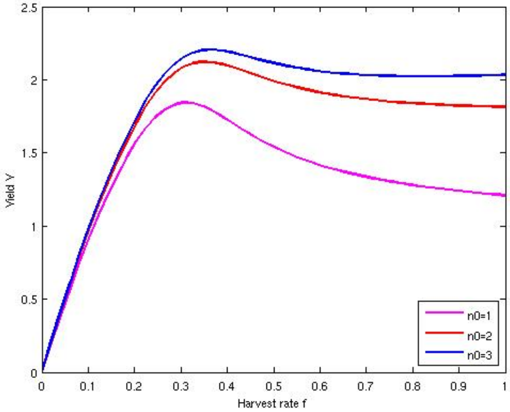
Yield as a function of harvest rate over various times:

- Initial population $n_0 = 1$

Simple logistic - transient (initial conditions)

Yield as a function of harvest rate for varying initial populations:

- Time $T = 10$



Problems with simple logistic model

Problem: Simple logistic model has no reproductive penalty

- Reproductive rate is the same regardless of population size

Solution: Add a square term to the model giving us skewed logistic model

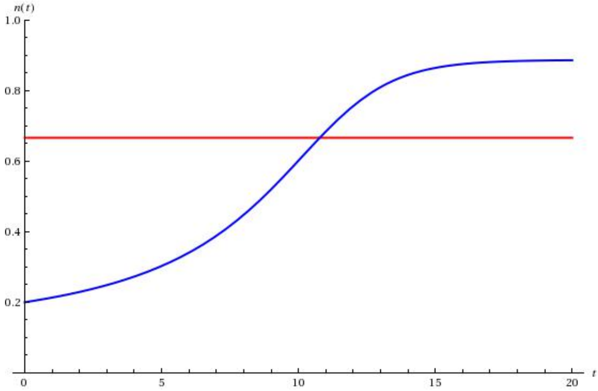
$$\frac{dn}{dt} = n^2(1 - n) - fn$$

Skewed logistic model - equilibrium

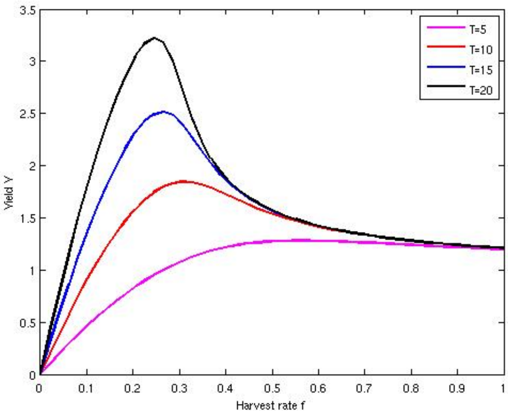
$$\frac{dn}{dt} = n^2(1-n) - fn$$

Solution:

- $n^* = \frac{2}{3}$
- $f^* = \frac{2}{9}$
- $Y^* = \frac{4}{27}$



Skewed logistic - transient (time)



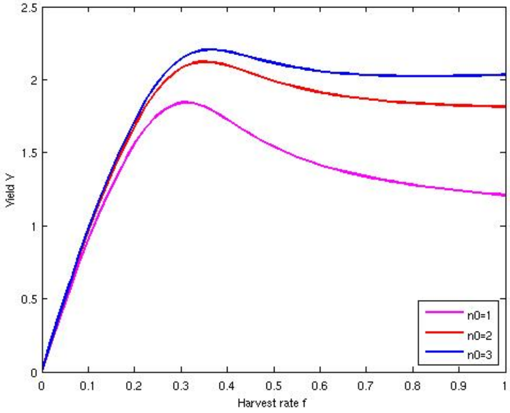
Yield as a function of harvest rate over various times:

- Initial population $n_0 = 1$

Skewed logistic - transient (initial conditions)

Yield as a function of harvest rate for varying initial populations:

- Time $T = 10$

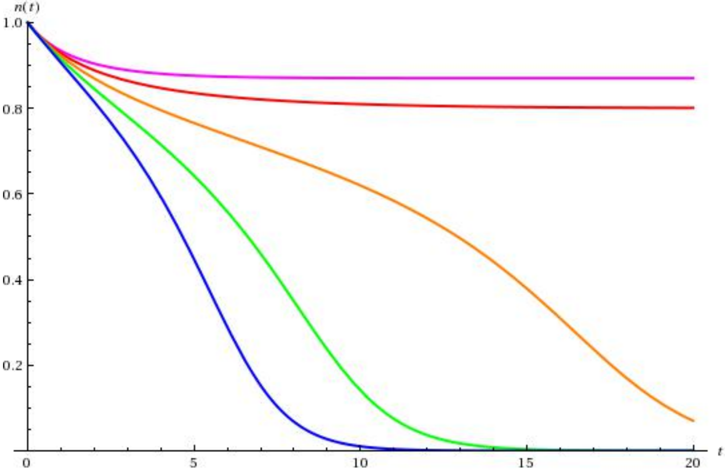


Allee effect model

How can we increase the penalty on reproduction?

- Add Allee effect
- As $0 < a < 1$ increases penalty on reproduction increases

Allee effect solution



Allee effect model - equilibrium

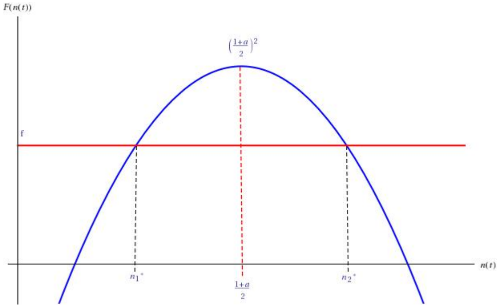
$$\frac{dn}{dt} = n(n-a)(1-n) - fn$$

There are two equilibria:

- n_1^* is unstable - population will go to extinction
- n_2^* is stable

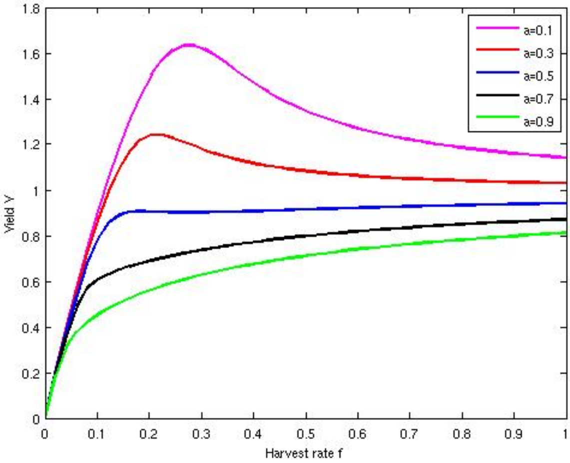
Allee effect model: $\frac{dn}{dt} = n(n-a)(1-n) - fn$

1. If $f \geq \left(\frac{1+a}{2}\right)^2$ as $t \to \infty$, $n(t)$ decreases to extinction
2. Given $f < \left(\frac{1+a}{2}\right)^2$
   - If $n_0 < n_1^*$, $n(t)$ decreases to extinction
   - If $n_0 > n_1^*$, $n(t)$ increases to n_2^*
3. Given n_0
   - If $f > (n_0 - a)(1 - n_0)$, $n(t)$ decreases to extinction
   - $\Rightarrow f \leq (n_0 - a)(1 - n_0)$

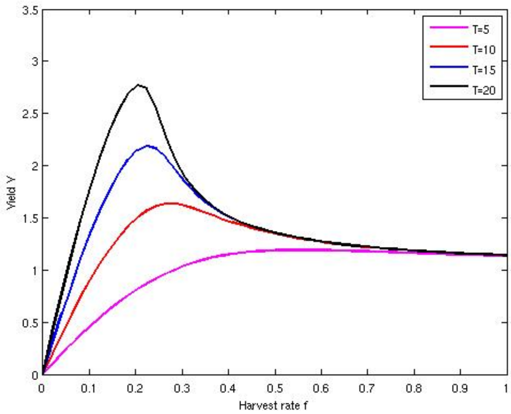


Allee effect model

Harvest vs. yield for various $0 < a < 1$



Allee effect - transient (time)



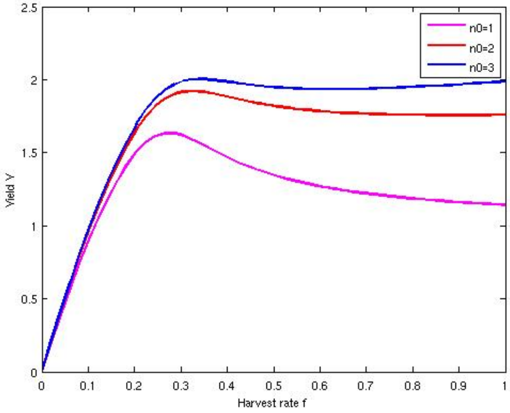
Yield as a function of harvest rate over various times:

- Allee effect $a = 0.1$
- Initial population $n_0 = 1$

Allee effect - transient (initial conditions)

Yield as a function of harvest rate for varying initial populations:

- $a = 0.1$
- Time $T = 10$



What role do initial conditions play?

Let $n(0) = n_0$ denote inital population density.

- *Allee effect* - if $n_0 < a$ the population goes extinct with constant harvesting.
- *Healthy population* - if the population is healthy, then constant and time dependent strategies give same yield.
- *Unhealthy* - if the population is unhealthy, ie. n_0 is small (less than equilibrium) then a time dependent harvesting will vastly increase yield over constant.

Possible further studies

1. Expand these models to a two stage population with juvenilles and adults
2. Construct models that include life history strategies of fish
3. Optimization problem with two stage harvest function
4. Implement harvest functions with more than two stages