Classifying Strictly Weakly Integral Modular Categories of Dimension 16p

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Categories

Category $\mathcal{C}$

- A class of objects $\text{Ob}(\mathcal{C})$
- A class of associative morphisms $\text{Hom}_{\mathcal{C}}(X, Y)$ between each pair of objects $X, Y \in \text{Ob}(\mathcal{C})$

Fusion Categories

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- Finite rank$\rightarrow$ Finitely many isomorphism classes of simple objects
- $\mathbb{1}$ is simple

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The **Müger center** of a braided fusion category $\mathcal{C}$ is defined

$$Z_2(\mathcal{C}) = \{X \in \mathcal{C} : C_{Y,X} \circ C_{X,Y} = \mathrm{id}_{X\otimes Y} \ \forall Y \in \mathcal{C}\}$$

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A **modular category** is a braided, spherical fusion category with trivial Müger center.

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- Determine fusion rules

$$X_i \otimes X_j = \sum N_{X_i,X_j}^{X_k} X_k$$

$$N_{X_i,X_j}^{X_k} = [X_i \otimes X_j : X_k]$$

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Definition

A simple object X is **invertible** if $\text{FPDim}(X) = 1$. Equivalently, $X \otimes X^* \cong \mathbb{1} \cong X^* \otimes X$.

Frobenius-Perron Dimension

$$\text{FPDim}(X \oplus Y) = \text{FPDim}(X) + \text{FPDim}(Y)$$

$$\text{FPDim}(X \otimes Y) = \text{FPDim}(X)\text{FPDim}(Y)$$

$$\text{FPDim}(X^*) = \text{FPDim}(X)$$

Integral and Weakly Integral Fusion Categories

A fusion category $\mathcal{C}$ is:

- **pointed** if $\mathrm{FPDim}(X_i) = 1$ for all simple $X_i \in \mathcal{C}$
- **integral** if $\mathrm{FPDim}(X_i) \in \mathbb{Z}$ for all simple $X_i \in \mathcal{C}$
- **weakly integral** if $\mathrm{FPDim}(\mathcal{C}) \in \mathbb{Z}$

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In a weakly integral modular category $\mathcal{C}$:

- $\text{FPDim}(X_i)^2 \big| \text{FPDim}(\mathcal{C})$ for all simple objects $X_i \in \mathcal{C}$
- $\text{FPDim}(X_i) = \sqrt{n}$ for some $n \in \mathbb{Z}^+$

Grading of a Fusion Category

Definition

A fusion category $\mathcal{C}$ is **graded** by a group G if:

- $\mathcal{C} = \oplus_{g \in G} \mathcal{C}_g$ for abelian subcategories $\mathcal{C}_g$
- $\mathcal{C}_g \otimes \mathcal{C}_h \subset \mathcal{C}_{gh}$ for all $g, h \in G$

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- A grading is called **faithful** if all $\mathcal{C}_g$ are nonempty.
- In a faithful grading, all components have dimension $\frac{\text{FPDim}(\mathcal{C})}{|G|}$
- If a simple object $X \in \mathcal{C}_g$, then $X^* \in \mathcal{C}_{g^{-1}}$
- $\mathcal{C}_e \supset \mathcal{C}_{ad}$, the smallest fusion subcategory containing $X \otimes X^*$ for all simple X

Grading of a Fusion Category

Universal Grading

- Every fusion category is faithfully graded by its universal grading group, $\mathcal{U}(\mathcal{C})$
- Every faithful grading is a quotient of $\mathcal{U}(\mathcal{C})$
- In a modular category, $\mathcal{U}(\mathcal{C}) \cong \mathcal{G}(\mathcal{C})$
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GN-Grading

- A weakly integral fusion category is faithfully graded by an elementary abelian 2-group E
- Simple objects are partitioned by dimension: For each $g \in E$, there is a distinct square-free positive integer n_g with $n_e = 1$ and $\text{FPDim}(X) \in \sqrt{n_g}\mathbb{Z}$ for all simple $X \in \mathcal{C}_g$
- $\mathcal{C}_e = \mathcal{C}_{int}$

Fusion Rules

For a simple object X,

$$X \otimes X^* \cong \mathbb{1} \oplus \bigoplus_{\substack{C_{ad} \ni y \ncong \mathbb{1} \\ y \otimes X \cong X}} y \oplus \bigoplus_{\substack{z \in \mathcal{C}_{ad} \\ |z| > 1}} N_{X,X^*}^{z} z$$

$\text{FPDim}(\mathcal{C}) = 16p$

- $\text{FPdim}(X_i) \in \{1, 2, 4, \sqrt{2}, 2\sqrt{2}, \sqrt{p}, 2\sqrt{p}, 4\sqrt{p}, \sqrt{2p}, 2\sqrt{2p}\}$ for all simple X_i
- $\sqrt{n_g} \in \{1, \sqrt{2}, \sqrt{p}, \sqrt{2p}\}$
- $|E| \in \{2, 4\}$

$\mathrm{FPDim}(\mathcal{C}) = 16p$, GN-Grading

dim	1	2	4	$\sqrt{2}$	$2\sqrt{2}$	$\sqrt{p}$	$2\sqrt{p}$	$4\sqrt{p}$	$\sqrt{2p}$	$2\sqrt{2p}$
# simples	a	b	c	f	d	h	k	l	m	n

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- $|\mathcal{C}_{pt}| \Big| |\mathcal{C}_{int}|$
- $|E| \Big| |\mathcal{U}(\mathcal{C})|$
- $|E| = 2 \to a \in \{4, 4p, 8, 8p\}$
- $|E| = 4 \to a \in \{4, 4p\}$

Example case: $|E| = 2, a = 8$

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- $|\mathcal{C}_g| = 2p = a_g + 4b_g + 16c_g \underset{4}{\equiv} 2$

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- $(\mathcal{C}_{ad})_{pt} = \{\mathbb{1}, g\} = \langle g \rangle \rightarrow \langle g \rangle$ is either modular or symmetric
- If $\langle g \rangle$ is symmetric, it is either sVec or Rep$(\mathbb{Z}_2)$

Case i: $B = \langle g \rangle$ is modular

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- $\mathcal{C} \cong B \boxtimes Z_{\mathcal{C}}(\mathcal{B})$
- $|Z_{\mathcal{C}}(\mathcal{B})| = 8p \rightarrow$ classified by Bruilliard, Plavnik, and Rowell

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If $\mathcal{D}$ is premodular and $\langle g \rangle = \text{sVec} \subset Z_{\mathcal{C}}(\mathcal{D})$, then $g \otimes X \not\cong X$ for all simple $X \in \mathcal{D}$.

- $\langle g \rangle = \text{sVec} \subset \mathcal{C}_{pt} = Z_{\mathcal{C}}(\mathcal{C}_{ad})$
- g stabilizes the simple objects of dimension 2 and 4 in $\mathcal{C}_{ad}$, a contradiction

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$\mathbb{Z}_2$-de-equivariantization of $\mathcal{C}$

- new fusion category $\mathcal{C}_{\mathbb{Z}_2}$ with $\text{FPDim}(\mathcal{C}_{\mathbb{Z}_2}) = \frac{\text{FPDim}(\mathcal{C})}{2}$
- for each simple $X \in \mathcal{C}$ such that $g \otimes X \cong X$, there are two simple objects in $\mathcal{C}_{\mathbb{Z}_2}$ with dimension $\frac{\text{FPDim}(X)}{2}$
- for each pair of simple objects $X \ncong Y$ such that $g \otimes X \cong Y$ (and $g \otimes Y \cong X$), there is one simple object in $\mathcal{C}_{\mathbb{Z}_2}$ with dimension $\text{FPDim}(X) = \text{FPDim}(Y)$

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The non-integral components of the universal grading of $\mathcal{C}$ have either

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- $h_g = 2$
- $m_g = 1$

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Simple objects of dimension $\sqrt{2}$ and $\sqrt{2p}$ are stabilized by g by parity. But $\frac{\sqrt{2}}{2}$ and $\frac{\sqrt{2p}}{2}$ cannot be the dimensions of simple objects in a fusion category. So the non-integral component of $\mathcal{C}$ must have simple objects of dimension $\sqrt{p}$.

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$|(\mathcal{C}_{\mathbb{Z}_2})_{int}| = 4p = 4 + 2b + 8c \to 2|b$
$|(\mathcal{C}_{\mathbb{Z}_2})_{pt}| \Big| |(\mathcal{C}_{\mathbb{Z}_2})_{int}| \to 4(1 + \frac{b}{2})|4p \to (b, c) \in \{(0, \frac{p-1}{2}), (2p-2, 0)\}$

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$\mathcal{C}_{ad}$ has only two invertibles, so there can be no simple objects of dimension 4 without simple objects of dimension 2.

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$\mathcal{C}_{\mathbb{Z}_2}$ is a generalized Tambara-Yamagami category:

Generalized Tambara-Yamagami Category

- non-pointed fusion category
- the tensor product of two non-invertible simple objects is a direct sum of invertible objects

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