Texas Geometry and Topology Conference

This is a report on the presentations at the 30th meeting of the Texas Geometry and Topology Conference at Rice University, October 31-November 2, 2003. This conference was partially supported by National Science Foundation Grant DMS-0306628 and Rice University. Speakers reported on recent research. For this report, speakers have provided brief synopses of their talks together with slightly broader discussions of the significance and context of their results.

Meeting 30. Rice University, October 31-November 2, 2003

Daniel Allcock, The University of Texas at Austin. *The fundamental group of the space of smooth cubic surfaces.*

We report joint work with J. Carlson and D. Toledo, proving that the fundamental group of the space of smooth cubi surfaces is not a lattice in any Lie group (that has finitely many connected components). This research was supported in part by NSF funding. The significance of the result is that the moduli space of smooth cubic surfaces cannot be realized as a quotient of any symmetric space by a discrete group. This means that an earlier result with Carlson and Toledo is the best possible, and provides one of the few known obstructions to a moduli space being realizable as a quotient of a symmetric space. This is important because each moduli space realizable this way seems to be an extraordinary exception requiring a special argument, but these exceptions seem to occur too often.

The main techniques of the proof are a nice mix of ideas: stratified Morse theory, elementary complex hyperbolic geometry and general theorems about lattices in Lie groups.

Richard W Kenyon, Université Paris-Sud and Princeton University. *Asymptotics of random discrete interfaces*

We discuss joint work with Andrei Okounkov. We study a model of random interfaces arising in the dimer model (domino tiling model). These are two-dimensional interfaces in R^3, and can be viewed as a higher-dimensional generalization of the simple random walk, where the domain is (part of) Z^2 instead of Z. We are interested in the “scaling limit” (limit when the mesh tends to zero). Specifically, there is a “law of large numbers” which says that at small mesh size a typical surface lies very close to its mean value. The mean value surface for given boundary values satisfies a variational principle (minimizing a certain energy functional), leading to a nonlinear PDE. Remarkably, its solutions can be parametrized by analytic functions (in a similar way as soap bubbles) but here one can see facets appearing in the limit shapes.

Peter Ozsv’ath, Columbia University. *Talk I: Floer homology and Dehn fillings*

This talk concerns joint work with Peter Kronheimer, Tom Mrowka, and Zoltan Szabo, in which we use methods from gauge theory to verify a conjecture of Gordon, according to which if p/q Dehn surgery on a knot is the lens space $L(p, q)$, then the knot is the unknot. The key technical device is a surgery long exact sequence for Seiberg-Witten monopole Floer homology. There are other applications of these techniques to problems of lens space surgeries, and also to the non-existence of taut foliations over certain three-manifolds.

Peter Ozsv’ath, Columbia University. *Talk II: Holomorphic disks, Seiberg-Witten monopoles, and low-dimensional topology*

This talk concerns recent results in low-dimensional topology, which are obtained from techniques in “Floer homology”. Specifically, I will discuss joint work with Zoltan Szabo, in which we construct such a Floer homology theory for three-dimensional manifold, by counting holomorphic disks in a complex manifold naturally associated to a Heegaard diagram. I will then turn to a completely different version of Floer homology (recently developed by Peter Kronheimer and Tomasz Mrowka), which counts “Seiberg-Witten monopoles” (solutions to a PDE coming from mathematical physics), but appears to give the same answer. I will then discuss some joint work with Kronheimer, Mrowka, and Szabo, in which we use monopole Floer homology to address some questions in low-dimensional topology.

Tony Pantev, University of Pennsylvania. ***Geometric transitions and integrable systems***

This is a report on a joint project in progress with E. Diaconescu, R. Donagi, B. Florea and A. Grassi. We study geometric transitions of Calabi-Yau manifolds from the point of view of the derived category of coherent sheaves. A geometric transition is a familiar process in algebraic geometry in which two components of the moduli of Calabi-Yau manifolds intersect along a common boundary. The typical situation involves a space M of CY manifolds all containing contractible rational curves and another space L parameterizing all smoothings of the singular CYs that one obtains after contracting the curves. We develop a non-linear version of a quantization argument used by Dijkgraaf-Vafa in the toric setup, to quantize the spaces of sheaves supported on the exceptional curves contained in the CYs in M. This allows us to reconstruct the space L directly from M without a reference to the geometric transition process. To achive this we specialize to a specific sublocus $S \subset M$ parameterizing Calabi-Yau manifolds containing a whole curve of singularities. Using the local geometry of M near S we show that when singularity type is constant along the curve and is of type A-D-E, the nested sequence of moduli spaces $S \subset M \subset L$ can be linearized by a rather subtle version of the deformation to the normal cone construction. Furthermore, the linearized moduli spaces admit interpretations as moduli spaces of (non-compact) linearizations of the original Calabi-Yau manifolds. Remarkably enough this process algebraizes the Hodge theory of the family L. Specifically we show that the analytic integrable system of intermediate Jacobians over L osculates to first order an algebraic integrable system over the linearization of L whose fibers are products of the intermediate jacobians of resolutions of CYs in S and Prym varieties which are fibers of a Hitchin system for the corresponding curve of singularities. Moreover, the structure group of the Hitchin system is precisely the A-D-E group labeling the type of singularities.

This gives us a geometric way of describing the Dijkgraaf-Vafa quantization procedure in a non-linear setup. It also allows us to compute the quantum superpotential asymptotically in a large variety of examples. The results have immediate applications to computing open Gromov-Witten invariants, non-commutative geometry, quantization of D-branes and matrix quantum mechanics.

Mihnea Popa, Harvard University. ***Asymptotic intersection numbers and restricted volumes***

The work described in my talk is related to one of the key problems in higher dimensional geometry, namely the existence of what is called a Zariski decomposition for divisors on a smooth projective variety. It was shown in pioneering work by O. Zariski and D. Mumford that such a decomposition exists on surfaces, and that it has fundamental consequences for understanding linear systems and for classification problems. Unfortunately on higher dimensional varieties such a decomposition does not exist. The purpose of the work I am describing is partly to develop methods which would overcome its absence.

Jason Starr, M.I.T. ***Sections of algebraic fibrations***

Systems of polynomial equations in several variables occur throughout mathematics and science. It frequently happens that the system of polynomial equations themselves depend on parameters that have some physical or mathematical meaning. The problem is to find, for each choice of the parameters, a solution of the resulting system of polynomial equations, preferrably a solution that depends in a simple way on the parameters. The best situation is if the solution has coordinates that are polynomials in the parameters, or perhaps ratios of polynomials. In different language, this is precisely the problem of finding a "k-rational point" of a variety defined over a non-algebraically-closed field. New research by several coauthors and myself guarantees "best situation" occurs if the system depends on only one parameter and satisfies a criterion, "rational connectedness", that is robust and often easy to check.

Given an algebraic (=holomorphic) map of complex projective varieties $f : X \to B$, what geometric condition on a general fiber of f guarantees that there is an algebraic section of f, or even a rational section of f? A related question is this: given a variety X defined over a non-algebraically-closed field k, what "geometric" condition guarantees that X has a "k-rational point"? The answer turns out to be closely related to a geometric condition called "rational connectedness". I will discuss some older results due to Tsen, Chevalley, Lang, etc., some newer results due to Graber, Harris, de Jong, Mazur and myself for the case $\dim(B) = 1$, and some conjectures for the case $\dim(B) > 1$ (on the way I will define rational connectedness and talk about some of its beautiful properties).

Genevieve Walsh, The University of Texas at Austin. ***Some examples of virtually Haken and virtually fibered three-manifolds***

We show that all two-bridge knot complements and many Montesinos knot complements are virtually fibered. We also show that infinitely many fillings of many two-bridge knot complements are virtually Haken.

This work addresses two open conjectures in three-manifold theory. The first is the virtually Haken conjecture, which asserts that every closed irreducible three-manifold with infinite fundamental group is either Haken or finitely covered by a Haken manifold. A Haken manifold is one that contains an incompressible surface. Much work has been done on this conjecture, and it has been proven under certain circumstances, although not for fillings of the knot complements we consider. The second is the virtually fibered conjecture, which asserts that every hyperbolic three-manifold is either fibered or finitely covered by a fibered manifold. There are few examples of manifolds that are virtually fibered but not fibered. Both of these are questions of Bill Thurston dating from the early 80's.

For any two-bridge knot complement, there is a finite cover that is the complement of a link of great circles in S^3. We show that for many two-bridge knots, this cover contains a closed incompressible surface. Infinitely many fillings of the two-bridge knot lift to fillings of great circle link where the incompressibility of this surface is preserved. Using this, we show that infinitely many fillings of an infinite class of two-bridge knot complements are virtually Haken. We call a knot a "spherical Montesinos knot" if the double branched cover of the three-sphere is a spherical Seifert fibered space. The complement of such a knot is finitely covered by a great circle link complement. We show that great circle link complements are fibered, which implies that all two-bridge knot complements and many Montesinos knot complements are virtually fibered.

Mu-Tao Wang, Columbia University. ***Mean curvature flows of Lagrangian submanifolds***

The mean curvature flow is the heat equation of submanifolds. A submanifold evolves in order to decrease its area as fast as possible along this process whose the stationary phase correspond to minimal submanifolds. A distinguished class of minimal submanifolds of Calabi-Yau manifolds are called special Lagrangians. Several important conjectures on Calabi-Yau manifolds demand deep understanding of the structure of special Lagrangians. However, so far there is no general procedure of constructing special Lagrangians. We propose to deform a Lagrangian submanifold to a special one by the mean curvature flow. The flow may develop singularities along the process. In order to complete the flow, we shall investigate the formation and surgeries of singularities.