Workshop in Linear Analysis and Probability
Abstracts of Talks - Seminar
Summer 2004

David Blecher, *Automatic w*-continuity and applications*

Functional analytic questions about spaces of operators often boil down to considerations involving dual, or weak*, topologies. In many such calculations, the key point is to prove that certain linear functions are weak* continuous. We offer a couple of results which ensure that a linear map be automatically continuous with respect to such topologies. For example, if X is a subspace of a C^*-algebra A, and if $a \in A$ satisfies $aX \subset X$ and $a^*X \subset X$, and if X is isometric to a dual Banach space, then we show that the function $x \mapsto ax$ on X is weak* continuous. In the right situation, such results can be extremely useful, as we show in many applications.

Tao Mei, *A transference trick in the analysis of functions on the unit circle*

We will explain a transference technique for functions defined on the unit circle, which permits to reduce many problems to the dyadic case. This transference technique works also for functions on $\mathbf{R}^n$.

Niels Nielsen, *Some operator $\mathcal{L}_p$ spaces*

In this lecture we shall investigate the local structure of non-commutative L_p-spaces and introduce some operator space analogues of the classical $\mathcal{L}_p$-spaces from Banach space theory. In particular we shall discuss operator spaces similar to the sequence spaces constructed by Rosenthal in 1970.

The lecture is based on the following two papers:

M. Junge, N.J. Nielsen, Z. Ruan, Q. Xu, The local structure of non-commutative L_p-spaces, to appear in Advances in Mathematics.

M. Junge and N.J. Nielsen, Rosenthal operator spaces, preprint.

Timur Oikhberg, *Operator ideals and order boundedness*

We consider the relation between positivity and order boundedness of operators between Banach lattices, on one hand, and operator ideals, on the other hand. Two questions of special interest are:

(1) Suppose (A, α) is a symmetrically normed ideal in $B(\ell_2)$. Then any member of A is regular if and only if A is a subset of S_2 (the space of Hilbert-Schmidt operators).

(2) Suppose A as above has the following property: if u and v are positive operators on ℓ_2, $u \leq v$, and $v \in A$, then $u \in A$. We show that such an A must contain S_2. On the other hand, if E is a separable sequence space which is an interpolation space between ℓ_2 and ℓ_∞, then the ideal S_E (consisting of all compact operators whose sequences of singular numbers belong to E) has this property.

We also mention similar problems regarding "general" Banach operator ideals.

CORRECTION: In the original abstract, I erroneously stated that, if E is a separable sequence space which is an interpolation space between ℓ_2 and ℓ_∞, $0 \leq u \leq v$, and $v \in S_E$, then $u \in S_E$. It has been pointed out to me that my proof contained an error. In fact, it was shown independently by V. Peller an B. Simon that, in case of $E = \ell_p$, the condition $0 \leq u \leq v \in S_E$ implies $u \in S_E$ if an only if p is an even integer.

David Opela, *Generalizing Ando's dilation theorem*

Any contraction on a Hilbert space H is a restriction of a co-isometry on a larger Hilbert space and a compression of a unitary (on a larger space). Ando's theorem says that any pair of commuting contractions is a restriction (compression) of a pair of commuting co-isometries (unitaries). An anologous result is not true for three commuting contractions, however, there are still ways to generalize...

Imre Patyi, *On smooth functions on a Banach space*

If a Banach space lacks smooth partitions of unity, then smooth functions on it exhibit a certain rigidness, which is shared also by, say, holomorphic functions. To study smooth vector bundles on open subsets of our Banach space it is useful to be able to resolve smooth cocycles with values in the number line or a Banach Lie group. The analog of this problem in several complex variables (Theorem B of Cartan, and the Grauert–Oka principle) provide us with useful methods to tackle this "smooth Cousin problem" in some Banach spaces that may lack a smooth partition of unity but in return satisfy a simple extension property for smooth self-maps, such as the spaces C(K) for K a compact metric space do. We show, e.g., that any smooth vector bundle that is continuously trivial over an open subset of, say, C[0,1], is smoothly trivial as well. We build on work by C.J. Atkin (New Zealand) and Lempert (Purdue).

Beata Randrianantoanina, *On extensions of Hölder maps with values in spaces of continuous functions*

Author: Gilles Lancien, Beata Randrianantoanina

Abstract: We study the isometric extension problem for Hölder maps from subsets of any Banach space into c_0 or into a space of continuous functions. For a Banach space X, we prove that any α-Hölder map, with $0 < \alpha \leq 1$, from a subset of X into c_0 can be isometrically extended to X if and only if X is finite dimensional. For a finite dimensional normed space X and for a compact metric space K, we prove that the set of α's for which all α-Hölder maps from a subset of X into $C(K)$ can be extended isometrically is either $(0, 1]$ or $(0, 1)$ and we give examples of both occurrences. We also prove that for any metric space X, the described above set of α's does not depend on K, but only on finiteness of K.

Narcisse Randrianantoanina, *Compact operators on C^*-algebras*

In this talk, I will discuss compactness of absolutely summing operators defined on C^*-algebras into general Banach spaces as extensions of the general theory of operators from $C(K)$-spaces. We show for example that 1-summing operators from $B(\ell_2)$ into spaces with RNP are compact.

Bunyamin Sari, *On a spreading model problem*

Consider the set of all spreading models of a Banach space generated by weakly null or block basic sequences equipped with the partial order defined by the usual domination of bases. In a work by G. Androulakis, E. Odell, Th. Schlumprecht and N. Tomczak-Jaegermann, it is proved that every countable subset of such spreading models admits an upper bound (in the set). In this talk we consider a problem posed by E. Odell. Suppose that a space admits a strictly increasing infinite chain of spreading models generated by weakly null sequences. Does it follow that the space admits an uncountable increasing chain of such spreading models?

Vladimir Troitsky, *Martingales in Banach lattices*

We present a version of martingale theory in terms of Banach lattices. A sequence of contractive positive projections (E_n) on a Banach lattice F is said to be a filtration if $E_nE_m = E_{n\wedge m}$. A sequence (x_n) in F is a martingale if $E_nx_m = x_n$ whenever $n \leq m$. Denote by M the Banach space of all norm uniformly bounded martingales. It is shown that if F doesn't contain a copy of c_0 or if every E_n is of finite rank then M is itself a Banach lattice. Convergence of martingales is investigated and a generalization of Doob Convergence Theorem is established. It is proved that under certain conditions there are isometric embeddings $F \hookrightarrow M \hookrightarrow F^{**}$. It is shown that every martingale difference sequence is a monotone basic sequence.

Marina Yaskina, *Modified Busemann-Petty problem on sections of convex bodies*

The Busemann-Petty problem asks whether origin-symmetric convex bodies in $\mathbf{R}^n$ with smaller central hyperplane sections necessarily have smaller n-dimensional volume. It is known that the answer is affirmative if $n \leq 4$ and negative if $n \geq 5$. We modify the assumptions of the original Busemann-Petty problem to guarantee the affirmative answer in all dimensions.