Workshop in Linear Analysis and Probability
Abstracts of SUMIRFAS Talks
Summer 2003

Razvan Anisca, *Unconditional Decompositions in Subspaces of* $\ell_2(X)$

If a Banach space X is not isomorphic to a Hilbert space then $\ell_2(X)$ contains a subspace which has a UFDD, but does not admit a UFDD with a uniform bound for the dimensions of the decomposition. We will also discuss some results about local unconditional structures in subspaces with UFDD.

Lawrence Fialkow, *Truncated multivariable moment problems and applications*

In work with R.E. Curto, we use positivity and extension properties of moment matrices to characterize the existence of a representing measure with the fewest possible atoms in the truncated multivariable K-moment problem, in the case when K is a semi-algebraic subset of R^d. In special cases, e.g., when K is a parabolic curve in the plane, we provide computable necessary and sufficient conditions for the existence of a K-representing measure and an algorithm for computing the K-representing measure with the fewest atoms. We discuss some applications to multivariable cubature (work with S. Petrovic) and applications to optimization theory due to J. Lasserre.

Miguel Martin, *Finite-dimensional Banach spaces with numerical index zero (joint work with J. Meri and A. Rodriguez-Palacios)*

We prove that a finite-dimensional Banach space X has numerical index 0 if and only if it is the direct sum of a real space X_0 and nonzero complex spaces $X_1, \dots, X_n$ in such a way that the equality

$$\left\|x_0 + \mathrm{e}^{iq_1\rho}x_1 + \cdots + \mathrm{e}^{iq_n\rho}x_n\right\| = \|x_0 + \cdots + x_n\|$$

holds for suitable positive integers $q_1, \dots, q_n$, and every $\rho \in \mathbb{R}$ and every $x_j \in X_j$ $(j = 0, 1, \dots, n)$. If the dimension of X is two, then the above result gives $X = \mathbb{C}$, whereas $\dim(X) = 3$ implies that X is an absolute sum of $\mathbb{R}$ and $\mathbb{C}$. We also give an example showing that, in general, the number of complex spaces cannot be reduced to one.

Ginés Löpez Përez, *Relatively weakly open subsets of the unit ball in functions spaces*

The nonexistence of denting points in the unit ball of some functions spaces has been the subject of several recent researching [3], [5]. A point x_0 in the sphere of a Banach space X, S_X, is a denting point of the unit ball in X, B_X, if there are slices, that is, subsets in the way

$$S(x^*, \alpha) = \{x \in B_X : x^*(x) > \|x^*\| - \alpha\},\ x^* \in X^*,\ \alpha \in \mathbb{R}\}$$

of diameter arbitrarily small. From [2], x_0 is a denting point of the unit ball of X if, and only if, x_0 is an extreme point in B_X and x_0 is a point of weak-norm continuity, that is, a point of continuity for the identity map from (B_X, w) onto (B_X, n), where w and n denote the weak and the norm topology, respectively. In particular, the existence of denting points in the unit ball of a Banach space X implies the existence of nonempty weak open subsets relative to the unit ball in X with diameter arbitrarily small. Then the extremely opposite property to the existence of denting points in the unit ball of a Banach space is the following: (property P) every nonempty weak open subset relative to the unit ball has diameter 2. This is the case, for example, for C^*-algebras [1], and uniform algebras [4].

The aim of this talk is to show some class of Banach spaces satisfying property P. For example, if X is a non-reflexive M-ideal in its bidual, it is showed that X and X^{**} verify property P. Also we characterize property P in operator spaces $L(X, C(K))$, $L(L_1(\mu), X)$, $K(X, C(K))$ (compact operators) and $WK(X, C(K))$ (weakly compact operators), where K is a compact space and X is a Banach space.

From the above, it is natural to think in the relation, up renorming, between property P and non PCP (point of continuity property). Given a Banach space X failing PCP, we don't know if it is possible renorming X to satisty property P.

References

[1] J. Becerra, G. López and A. Rodríguez. *Relatively weakly open sets in closed balls of C^*-algebras.* Preprint. To appear in J. London Math. Soc.

[2] Bor-Luh Lin, Pei-Kee Lin and S. L. Troyanski. *Characterizations of denting points*. Proc. Amer. Math. Soc. 102, (1988), 526-528.

[3] Z. Hu and M. A. Smith. *On the extremal structure of the unit balls of Banach spaces of weakly continuous functions and their duals.* Trans. Amer. Math. Soc. 349, (1997), 1901-1918.

[4] O. Nygaard and D. Werner. *Slices in the unit ball of a uniform algebra.* Archiv Math. 76, (2001), 441-444.

[5] T. S. S. R. K. Rao. *There are no denting points in the unit ball of $WC(K, X)$.* Proc. Amer. Math. Soc. 127, (1999), 2969-2973.

Matt Neal, *JB*-triples in Operator Space Theory*

In recent papers by Russo and the author, JB*-triple theory has been shown to have natural applications to certain questions in operator space theory. The greater part of the talk will focus on the result that all finite dimensional Hilbertian subspaces of a C*-algebra (or TRO) which are contractively complemented are (with a natural qualification) completely isometric to actions of the underlying Hilbert space on the anti-symmetric Fock space. Various other results will be given about the operator space structure of these spaces and the infinite dimensonal problem will be discussed. Other applications of JB*-triples include recent operator space classifications of TRO's, Left ideals, and C*-algebras via Banach space classifications of JB*-triples, and the general classification of contractively complemented operator spaces.

Timur Oikhberg, *Representing Banach algebras as algebras of completely bounded maps*

We prove that, if A is a weak* closed subalgebra of $B(\ell_2)$, then there exists an operator space structure X on $\ell_2 \otimes \ell_2$ such that $CB(X) = A \otimes I_{\ell_2} + S_2$ (S_2 is the space of Hilbert-Schmidt operators). We present a generalization of this result to Banach algebras. Finally, we construct examples of (1) a Hilbertian operator space X completely isomorphic $X \oplus X \oplus X$, but not $X \oplus X$; (2) a Hilbertian operator space Y s.t. $K_1(CB(Y))$ contains torsion elements.

Bernie Russo, *State spaces of JB*-triples*

An atomic decomposition is proved for Banach spaces which satisfy some affine geometric axioms compatible with notions from the quantum mechanical measuring process. This is then applied to yield, under appropriate assumptions, geometric characterizations, up to isometry, of the unit ball of the dual space of a JB*-triple, and up to complete isometry, of one-sided ideals in C*-algebras.

Gideon Schechtman, *Integral orthogonal splittings of* L_1^{2k}

For each positive integer k there is a $k \times k$ matrix B with ± 1 entries such that letting E be the span of the rows of the $k \times 2k$ matrix $[\sqrt{k} I_k, B]$, $E, E^\perp$ is a Kashin splitting: The L_1^{2k} and the L_2^{2k} are universally equivalent on both E and $E^\perp$. Moreover, the probability that a random ± 1 matrix satisfies the above is exponentially close to 1. This together with a formula of Anderson hints at where to look for an explicit Kashin splitting.

Roger Smith, *Perturbations of Subalgebras of von Neumann Algebras*

In this talk we will consider the relationship between two subalgebras of a type II_1 factor when they are close to one another in a metric which we will introduce. The main result is that the structure is quite rigid, in the sense that the algebras can be cut by large projections (meaning that we throw away small parts) so that the remaining parts are spatially isomorphic by a partial isometry close to 1. We will give examples and background material so that little will be assumed. Time permitting, we will also discuss some applications to the topic of maximal abelian self-adjoint subalgebras (masas). This is joint work with Sorin Popa and Allan Sinclair.

Przemek Wojtaszczyk, *Projections and nonlinear approximation in the space $BV(\mathbb{R}^d)$*

In this talk we want to present results about non-linear approximation in the $L_p$norm $p = d/(d-1)$ of functions of bounded variation on $\mathbb{R}^d$ with $d > 1$ bypolynomials in the Haar system. The exponent p is the natural exponent as it is the correct exponent in the Sobolev inequality. The space of functions of bounded variation turned out to be quite useful in geometric measure theory, calculus of variation and differential equations. More recently (but really only in the case $d = 2$) this space was used in image processing. This results from the idea that a "natural" black and white image consists of regions of smooth changes of density and sharp jumps on the boundaries between regions. Such densities are in fact functions of bounded variation.

The approximation schemes we are discussing in this talk are mostly related to Haar tresholding and m–term approximation. This technique of decomposing the signal in Haar wavelet (or other wavelet) basis and choosing few most significant summands is widely used in practical image compression algorithms.

Those problems for $d = 2$ were studied in detail in the paper by A. Cohen, R. DeVore, P. Petrushev and H. Xu, *Nonlinear approximation and the space $BV(\mathbb{R}^2)$*, Amer. J. Math. 121(1999) pp.587-628. The study of the case $d > 2$ is justified by simple mathematical curiosity but also by the hope of understanding three dimensional signals and the fact that minimization of various functionals related to variation (also for dimension > 2) plays an important role in problems of differential equations. The ingredients in our analysis are

1. boundedness in BV($\mathbb{R}^d$) of certain class of averaging operators.

2. nonlinear approximation in L_p. For this we use (variants of) recent results on greedy approximation with respect to the Haar system

This talk is based on my paper *Projections and non-linear approximation in the space $BV(\mathbb{R}^d)$* which is to appear in Proc. London Math. Soc.

Guoliang Yu, *Uniform Convexity and Novikov type conjectures*

In this talk, I will explain how uniform convexity of Banach spaces can be used to study Novikov type conjectures. This talk will be expository and should be accessible to non-expert. This is joint work with Gennadi Kasparov

Vrej Zarikian, *The Calculus of One-sided M-Ideals in Operator Spaces*

This talk will survey recent work of the author with David Blecher on one-sided M-ideals and multipliers in operator spaces. We will begin by recalling the classical M-ideal theory of Banach spaces, introduced and developped by Alfsen and Effros in 1972. In order to set the stage for our non-commutative generalization of this theory, we then explain the connection between classical M-ideals and Banach space multipliers and centralizers. After a brief discussion of operator space generalities, we explain Blecher's theory of one-sided multipliers of operator spaces, emphasizing recent contributions of Blecher, Effros, and the author. Finally, we use one-sided multipliers to define and investigate one-sided M-ideals in operator spaces. The talk will be suitable for a general functional analysis audience.

Artem Zvavitch, *Projections of convex bodies and analytic characterizations of zonoids*

In this talk we present the Fourier analytic approach to projections of convex bodies based on a formula expressing the volume of hyperplane projections in terms of the Fourier transform of the curvature function. (This is a joint work with A. Koldobsky and D. Ryabogin)