# Open problems raised during the Workshop in Analysis and Probability

July-August 2008

The problems here were either submitted specifically for the purpose of inclusion in this list, or were taken from talks given during the Workshop in Linear Analysis and Probability.

## 1 Seminars

**Problem 1.** (Submitted by Yun-Su Kim)
Let $V$ and $W$ be abstract operator spaces. Is $CB(V,W)$ dense in $N^p(V,W)$?

**Problem 2.** (Submitted by Bentuo Zheng)
Let $1 < p < \infty$ and $X$ be a separable reflexive Banach space. Assume that $X$ satisfies an asymptotic lower-$\ell_p$-tree estimate, and that $T :\to Y$ is a bounded linear operator that satisfies an asymptotic upper-$\ell_p$-tree estimate. Does $T$ factor through a subspace of a reflexive space with an asymptotic $\ell_p$ FDD?

**Problem 3.** (Submitted by Rachid El Harti)
Let $A$ be any non-simple $C^*$-algebra. Is there a non-trivial pro-$C^*$-algebra structure for $A$?

**Problem 4.** (Submitted by Rachid El Harti)
Let $A$ be a non-trivial, non-commutative unital pro-$C^*$-algebra. Is there a non-unital $C^*$-algebra $A_0$ such that $A$ is the multiplier algebra of $A_0$?

**Problem 5.** (Submitted by Rachid El Harti)
Let $A$ be a pro-$C^*$-algebra that is the bounded part of the inverse limit of a system of $C^*$-algebras $(A_\alpha)_{\alpha\in I}$. If each $A_\alpha$ is RR0, is $A$ necessarily also RR0?

**Problem 6.** (Submitted by Hector Salas)
Let $H$ be an infinite dimensional Hilbert space, and let $\mathcal{H}_H$ denote the subclass of $\mathcal{L}(H)$ consisting of the hypercyclic operators on $H$. Let $\sigma \subset \mathbb{C}$ be compact such that each component intersects the unit circle $\mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$. Must there exist $T \in \mathcal{H}_H$ such that its spectrum $\sigma(T) = \sigma$? More generally, for each infinite dimensional separable Banach space $X$ characterize those $\sigma$ for which there exists $T \in \mathcal{H}_X$ with $\sigma = \sigma(T)$.

**Problem 7.** (Submitted by Hector Salas)
Let $X$ be an infinite dimensional separable Banach space such that its dual $X^*$ is also separable. An operator $T \in \mathcal{H}_X$ is dual hypercyclic if $T \in \mathcal{H}_{X^*}$ (such operators exist). Identify the compact subsets of $\mathbb{C}$ which are the spectra of dual hypercyclic operators.

**Problem 8.** (Submitted by Hector Salas)
Let $X$ be a topological space such that multiplication is a continuous mapping $\mathbb{T} \times X \mapsto X, (\lambda, x) \mapsto \lambda x$ with $1x = x$ and $(\lambda\mu)x = \lambda(\mu x)$. What are the conditions on $X$ and $T : X \mapsto X$ continuous for which $\lambda T$ has a dense orbit for each $\lambda \in \mathbb{T}$? Although the question so posed is quite vague, a particularly interesting case is the infinite torus $X = \mathbb{T} \times \mathbb{T} \times ...$

**Problem 9.** (Submitted by Simon Cowell)
For a separable Banach space $X$, $X$ has $(au^*)$ implies that $X$ has $(au)$. Under what extra hypotheses on $X$ are they equivalent? In particular, are they equivalent provided that $X$ does not contain $\ell_1$?

**Problem 10.** (Submitted by Deping Ye)
Let the Hilbert space to be $H = \mathbb{C}^2 \otimes \mathbb{C}^2$. What is the exact Hilbert-Schmidt volume of separable states on $H$? What is the exact Bures volume of Separable states on $H$?

**Problem 11.** (Submitted by Stephen Dilworth)

(a) For which smooth Banach spaces $X$ do the XGA and/or the DGA converge?

(b) Do these algorithms converge in uniformly smooth spaces?

(c) Does the XGA converge in $L_p[0, 1]$ (or even $\ell_p$) for $1 < p < \infty$, $p \neq 2$?

(d) Does the XGA converge in $L_p[0,1]$ $(1 < p < \infty, p \neq 2)$ when the dictionary is the Haar system?

(see *Weak convergence of greedy algorithms in Banach spaces* by S. J. Dilworth, Denka Kutzarova, Karen Shuman, V. Temlyakov, and P. Wojtaszczyk, to appear in J. Fourier Anal. Appl.)

**Problem 12.** (Submitted by Piotr Nowak)
Let $G$ be a finitely generated group. Knowing that the fundamental class $[G]$ vanishes in $H_0^f(G)$ for $f$ of growth slower than linear, does there exist an aperiodic tiling of $G$? (see J.Block, S.Weinberger, JAMS 1992 5 (4) pp. 907-918. for the case $f = \text{const}$).

**Problem 13.** (Submitted by Piotr Nowak)
Let $G$ be a finitely generated group. Is there a growth type $\phi$ sufficiently slow such that for any $G$, if the fundamental class vanishes in $H_0^f(G)$ for $f$ slower than $\varphi$ then it vanishes in $H_0^f(G)$ for $f = \text{const}$?

**Problem 14.** (Submitted by Brett Wick)
Give an intrinsic characterization of the set of functions $f$ such that $d\mu_f = |f|^2 dA$ is $\mathcal{D}$-Carleson.

**Problem 15.** (Submitted by Antoine Flattot)
Does the Bishop operator have an invariant subspace for every irrational number?

**Problem 16.** (Submitted by Miguel Martín)

1. Find more sufficient conditions for a set to be SCD. For instance, if $X$ has a 1-symmetric basis, is $B_X$ an SCD set?

2. Let $E$ be a Banach space with unconditional basis. Is $E$ SCD?

3. Let $X$ and $Y$ be SCD spaces. Are $X \otimes_\varepsilon Y$ and $X \otimes_\pi Y$ SCD?

4. If $T_1$ and $T_2$ are SCD operators, is $T_1 + T_2$ SCD?

5. If $T : X \to Y$ is an SCD operator, is there an $SCD$ space $Z$ such that $T$ factors through $Z$?

## 2 Concentration week on multidimensional operator theory

**Problem 17.** (Submitted by Stefan Richter)
Characterize the extremals for the families of $d$-commuting contractions, $d$-contractions and $d$-isometries.

**Problem 18.** (Submitted by Lawrence Fialkow)

(a) For the truncated moment problem in the plane (2 real variables), it is known that if the degree of the problem is 2, then there is a representing measure if and only if the moment matrix for the data, $M(1)$, is positive semidefinite. If the degree is 6, it is known that there are examples where the moment matrix, $M(3)$, is positive definite, but there is no measure. If the degree is 4 and the moment matrix, $M(2)$, is positive definite, is there a representing measure?

(b) Suppose $p(x, y)$ is a real polynomial with $\deg p(x, y) = 2$, and suppose the restriction of $p$ to the closed disk, $p|D$, is positive. It can be shown that $p$ is of the form

$$p = r + (1 - x^2 - y^2)s, \tag{*}$$

where $r$ and $s$ are each sums of squares of polynomials of degree at most 1. It is known that for $N > 0$, there are polynomials $p$ of degree 6, with $p|D$ positive, such that in any representation of $p$ as in (*), the degree of some summand in the sums of squares has degree $> N$. Does there exist $M > 0$, such that if $\deg p = 4$ and $p|D$ is positive, then $p$ admits a representation (*) where each summand has degree at most $M$?

## 3 Concentration week on Operator Algebras, Dynamics and Classification

**Problem 19.** (Submitted by Søren Eilers)

(a) Understand why the $K$-web is sufficient for classifying Cuntz-Krieger algebras in spite of their ideal lattice being non-linear.

(b) Augment the invariant to arrive at a complete classification of purely iinfinite $C^*$-algebras with finitely many ideals.

(c) Classify the sofic $S$-gap shifts.

**Problem 20.** (Submitted by Chris Phillips)

(a) Let $D \subseteq A_0 \subseteq A_1 \subseteq \dots$. Assume that $D$ is a MASA in $A_n$ for all $n$. Doest it follow that $D$ is a MASA in the direct limit of the $A_n$'s?

(b) What is $\Gamma_{M_n}(D_n)$ (i.e. the commutation constant of the $n \times n$ diagonal matrices in the $n \times n$ matrices)?

(c) Let $A$ be a unital $C^*$-algebra, and let $D \subseteq A$ be a "good" MASA. Does it follow that $\Gamma_A(D) = 1/2$? Does it follow that $\Gamma_A(D) \leq 1$?

**Problem 21.** (Submitted by Jesse Peterson)
If $L(\Gamma_1) \simeq L(\Gamma_2)$, $C^*_r(\Gamma_1) \simeq C^*_r(\Gamma_2)$ then do we have that $\beta^(2)_1(\Gamma_1) = 0$ if and only if $\beta^(2)_1(\Gamma_2) = 0$?

**Problem 22.** (Submitted by David Sherman)

(a) (Dixmier) Let $\pi, \rho : A \to \mathcal{B}(H)$ be two representations of a $C^*$-algebra such that $\phi(x)$ is unitarily equivalent to $\rho(x)$ for all $x$ in $A$. Are $pi$ and $rho$ equivalent?

(b) Does locally inner imply inner for a separable $C^*$-algebra $A$?

(c) Are the concepts of $\kappa$-local innerness different for $C^*$-algebras?

**Problem 23.** (Submitted by Ilijas Farah)

(a) Assuming CH or Martin's axiom, there exist an ultrafilter $\mathcal{U}$ such that $F_{\mathcal{U}}(\mathcal{B}(H)) = \mathbb{C}$. Can we remove the axioms?

(b) Does there exist a nonprincipal ultrafilter $\mathcal{V}$ on $\mathbb{N}$ such that $F_{\mathcal{V}}(\mathcal{B}(H)) = \mathbb{C}$?

(c) What can be said about the structure of $F_{\mathcal{V}}(\mathcal{B}(H))$ in general?

(d) Is $F_{\mathcal{U}}(\mathcal{B}(H)) \neq \mathbb{C}$ equivalent to $\mathcal{V}$ is flat?

**Problem 24.** (Submitted by Narutaka Ozawa)

(a) Is there a von Neumann algebra that fully remembers the group action?

(b) Is there a von Neumann algebra that fully remembers the group?

(c) Is $vN(\mathbb{F}_r) \ncong vN(\mathbb{F}_s)$ for $r \neq s$?

For further submissions or corrections, send an email to
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