Elliptic Macdonald functions of BC type and q-Series Identities

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Abstract: Finding explicit bases for the ring of symmetric polynomials has a long history, stimulated by problems in algebraic geometry, group representation theory and harmonic analysis, probability, and algebraic combinatorics. The most famous classical example are the Schur functions. H. Jack, P. Hall, and D. E. Littlewood discoverd the first important extensions of the Schur functions, by introducing an extra parameter, called respectively the Jack and the Hall-Lttlewood polynomials. Ian Macdonald then obtained a joint generalization of these families, involving two parameters, which are called the Macdonald polynomials. In the last two decades there has an explosion of interest in these polynomials. Macdonald later defined analogs of these symmetric polynomials associated (essentially) to other root systems (besides A_n, the original case). We discuss a (five parameter) family of abelian functions in several variables generalizing the Macdonald polynomials (of A_n type), which are defined via a combinatorial ("branching") rule extending the classical combinatorial (tableaux) definition for the Schur functions. These functions are used to prove multivariate extensions of Frenkel and Turaev's elliptic hypergeometric identities discovered in connection with the elliptic Yang-Baxter equation. We also consider some applications to multivariate q-series identities extending, for example, Euler's pentagonal number theorem.