Texas A&M Algebra/Combinatorics Seminars
November 19, 20, 21

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Title: *A Generalization of Zaremba's Conjecture*

Wednesday November 19th, 4pm, Milner 317

Abstract: We ask, for which n does there exists a k, $1 \leq k < n$ and $(k, n) = 1$, so that k/n has a continued fraction whose partial quotients are bounded in average by a constant B? This question is intimately connected with several other well-known problems, bridging the theory of quasirandomness, discrepancy theory, exponential sums, and diophantine approximation. We discuss these relationships, prove a lower bound in the case of $B = 2$ using an elementary "shifting" argument, and present several intriguing questions in this area.

Title: *Quasirandomness and Fourier Coefficients*

Thursday November 20st, 10am, Milner 317

Abstract: Randomness plays an integral role in modern combinatorics, particularly when constructions are desired. However, it is often unsatisfying to call a randomly selected object a "construction", as no explicit description is given. This problem is notorious throughout Ramsey theory, number theory, and elsewhere. Since probabilistic methods have been so successful, it would be desirable to provide constructions that resemble random ones in salient ways – and, furthermore, to be able to "measure the randomness" of an object or class of objects. In fact, such measures have wide applications in structural combinatorial theory.

Quasirandomness is one such perspective on quantifying randomness. The set of properties true with high probability in a combinatorial space are partially ordered by implication; equivalence classes of logically equivalent properties may be identified and given the induced ordering. Surprisingly large sets of natural random-like properties have been shown to belong to a single one of these property cliques in the case of graphs, hypergraphs, tournaments, and subsets of the finite cyclic groups. In this talk, we discuss the theory of quasirandom permutations, a subject that relates discrepancy theory, permutation statistics, and finite harmonic analysis. Almost all natural number-theoretic permutations turn out to be highly quasirandom, and determining the relevant bounds provides a wealth of interesting questions in the theory of exponential sums. We discuss these and other examples, including the class of "optimally" quasirandom permutations, and we present several fundamental questions in this realm.

Title: *De Bruijn Covering Codes*

Friday November 21st, 3pm, Milner 317

Abstract: A de Bruijn covering code is a q-ary string S so that every q-ary string is at most R symbol changes from some n-word appearing consecutively in S. We introduce these codes and prove that they can have length close to the smallest possible covering code. The proof employs tools from field theory, probability, and linear algebra. We discuss specific bounds for small binary de Bruijn covering codes and highlight several important open questions.