HALF-FILLING FAMILIES OF FINITE SETS

ANDRÁS ZSÁK

Let $\mathcal{F}$ be a collection of finite subsets of a set X. We say that $\mathcal{F}$ is *hereditary* if $B \in \mathcal{F}$ whenever $B \subset A \in \mathcal{F}$; we say $\mathcal{F}$ is *half-filling* if it is hereditary and for every finite set $S \subset X$ there is a set $A \in \mathcal{F}$ such that $A \subset S$ and $|A| \geq \frac{1}{2}|S|$. The following Ramsey-type problem is still open: given a half-filling family on ω_1 (the first uncountable ordinal) is there an infinite subset of ω_1 all whose finite subsets belong to $\mathcal{F}$?

It can be shown that a negative answer would imply that for every countable ordinal α there is a minimal compact half-filling family on $\mathbb{N}$ with Cantor-Bendixon index at least α. Our main result is that this consequence of a negative answer is in fact true.

Date: April 25, 2004.