Approximate Solutions of the Incompressible Euler Equations with no Concentrations

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Abstract

We present a sharp local condition for the lack of concentration (and hence – the L^2 convergence of) sequences of approximate solutions to the incompressible Euler equations. Our approach, based on an H^{-1} stability condition, relies on using a generalized Div-Curl Lemma to replace the role that elliptic regularity theory has played previously in this problem. We apply this characterization to greatly simplify known existence results for 2D flows in the full plane (— with special emphasize on rearrangement invariant regularity spaces), and obtain new existence results of solutions without energy concentrations in any number of spatial dimensions.

Our results identify the 'critical' regularity which prevent concentration, regularity which is quantified in terms of Lebesgue, Lorentz, Orlicz and Morrey spaces. In particular, the strong convergence criterion cast in terms of circulation logarithmic decay rates due to DiPerna & Majda is simplified (— removing the weak control of the vorticity at infinity) and extended (— to any number of space dimensions).

Finally, we introduce a new scale of intermediate function spaces covering the gap between the weak $L^{p\infty}$ spaces and the larger Morrey spaces, and allowing us to make the subtle distinctions in the N-dimensional borderline cases which separate between H^{-1}-compactness and the phenomena of concentration-cancelation. Expressed in terms of the new scale of spaces, these borderline cases are shown to be intimately related to uniform bounds of the coulomb energy and the configuration of the vorticity.