Computational Electromagnetics I
An Introduction to Maxwell's Equations with 2D Applications

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The first lecture starts with an introduction to the Maxwell equations. The Maxwell equations built the basis for the computer simulation of electromagnetic fields that becomes more and more important not only in electrical engineering but also in many other disciplines including live sciences. During the last decade mathematicians have made important contributions to the correct modelling, the analysis and the efficient numerical solution of the electromagnetic field equations.

In order to simplify the presentation in our first lecture, we only consider the magnetostatic case in 2D that already allows us to describe with a reasonable accuracy interesting practical problems such as electrical machines. In the 2D magnetostatic case the Maxwell equations can be reduced to a potential equation that is in general non-linear. For bounded computational domains, we developed and implemented an optimal order finite element multigrid solver. The combination of this multigrid solver with a nested inexact Newton iteration results in a very efficient iteration scheme. However, in the general, the computational domain is unbounded. So, we focus our investigation on the construction of an optimal order solver for coupled finite and boundary element schemes based on a symmetric domain and boundary integral variational formulation of the corresponding (linear) potential equation. This approach can be extended to a general non-overlapping domain decomposition scheme that provides the basis for the construction of highly efficient parallel solvers. The preconditioner for the resulting symmetric, but indefinite system of algebraic equations is certainly the most important part of solver. The components for building such preconditioners can be borrowed from both the finite and the boundary element worlds.

Finally, we look at some numerical results that nicely confirm our theoretical predictions.