Families of sections of a convex body

It is a consequence of a result due independently to C. Bessaga (1958) and to B. Grűnbaum (1958) that for every pair of positive integers k, n with $1 < k < n$ there exists a finite family of centrally symmetric convex bodies in $\mathbb{R}^k$ whose affine images cannot be central sections of the same centrally symmetric convex body in $\mathbb{R}^n$. The smallest cardinality $N = N(k, n)$ of such a family is unknown. V. Klee (1960) asked, in a different language, what happens if one considers families of k-dimensional central sections of the unit ball of the space ℓ^p for $1 \leq p < \infty$. The answer turns out to depend on the arithmetic properties of p. It is related to the following well known problem: Given an even integer p find the smallest positive integer $h = h(k, p)$ such that the unit ball of the space ℓ^p_h has a k-dimensional central section which is an ellipsoid.
The proofs of the results involve various techniques from topology, dimension theory, harmonic analysis, convexity, and local theory of Banach spaces.
More recent material is taken from: F. Delbaen, H. Jarchow, A. Pełczyński, Subspaces of L_p isometric to subspaces of ℓ_p, Positivity 2 (1998), 339-367.