Traces of functions with an application to Banach space theory of Sobolev spaces

The trace of a continuous function on a subset of the domain of the function is the restriction of the function to that subset. It is more delicate to define the trace of measurable function on a set of measure zero. This kind of problems naturally appear when considering boundary values of analytic and harmonic functions and generalized solutions of certain partial differential equations. A simple example of a result in this spirit is the fact that a function from a Sobolev space $L_1^p(\mathbb{R}^n)$ for $n > 1$ has on every $n-1$-dimensional hyperplane H a trace which is a function in $L^p(H)$ with respect to the $n-1$-dimensional Lebesgue measure. In the case $p = 1$ Gagliardo (1957) noticed that every function in $L^1(H)$ is a trace of some function from $L_1^1(\mathbb{R}^n)$. Thus the trace is a linear surjection from $L_1^1(\mathbb{R}^n)$ onto $L^1(H)$. Peetre (1979) proved that this surjection has no right inverse. Applying these facts one can show that for Sobolev spaces $L_k^1(\mathbb{R}^n) \quad (n = 2, 3, \ldots; k = 1, 2, \ldots)$ there is no analogue of the Lebesgue Decomposition Theorem. More precisely the Sobolev space $L_k^1(\mathbb{R}^n)$ is uncomplemented in its second dual. This implies that $L_k^1(\mathbb{R}^n)$ is not a Banach lattice in any equivalent norm.
The proofs use classical analysis and Banach space theory.
New results are taken from: A. Pełczyński and M. Wojciechowski, Sobolev spaces in several variables in L^1-type norms are not isomorphic to Banach lattices.