

Erdős Magic

The Probabilistic Method is a lasting legacy of the late Paul Erdős. We give two examples - both problems first formulated by Erdős in the 1960s with new results in the last few years and both with substantial open questions. Further in both examples we take a Computer Science vantagepoint, creating a probabilistic algorithm to create the object (coloring, packing respectively) and showing that with positive probability the created object has the desired properties.

- Given m sets each of size n (with an arbitrary intersection pattern) we want to color the underlying vertices Red and Blue so that no set is monochromatic. Erdős showed this may always be done if $m < 2^{n-1}$, we give a recent argument of Srinivasan and Radhakrishnan that extends this to $m < c2^n \sqrt{n/\ln n}$. One first colors randomly and then recolors the blemishes with a clever random sequential algorithm.
- In a universe of size N we have a family of sets, each of size k , such that each vertex is in D sets and any two vertices have only $o(D)$ common sets. Asymptotics are for fixed k with $N, D \rightarrow \infty$. We want an asymptotic packing, a subfamily of $\sim N/k$ disjoint sets. Erdős and Hanani conjectured such a packing exists (in an important special case of asymptotic designs) and this conjecture was shown by Rödl. We give a simple proof of the speaker that analyzes the random greedy algorithm.