Experimentation in Mathematics: Computational Paths to Discovery, I and II

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Abstract. *In the first of two lectures I shall talk generally about experimental mathematics. In the second part, I aim to present some more detailed and sophisticated examples.*

The emergence of powerful mathematical computing environments, the growing availability of correspondingly powerful (multi-processor) computers and the pervasive presence of the internet allow for research mathematicians, students and teachers, to proceed heuristically and 'quasi-inductively'. We may increasingly use symbolic and numeric computation visualization tools, simulation and data mining.
Many of the benefits of computation are accessible through low-end 'electronic blackboard' versions of experimental mathematics [1, 7]. This also permits livelier classes, more realistic examples, and more collaborative learning. Moreover, the distinction between computing (HPC) and communicating (HPN) is increasingly moot.
The unique features of our discipline make this both more problematic and more challenging. For example, there is still no truly satisfactory way of displaying mathematical notation on the web; and we care more about the reliability of our literature than does any other science. The traditional role of proof in mathematics is arguably under siege. Limned by examples, I intend to pose questions ([8]) such as:

- What constitutes secure mathematical knowledge?
- When is computation convincing? Are humans less fallible?
- What tools are available? What methodologies?
- What about the 'law of the small numbers'?
- How is mathematics actually done? How should it be?
- Who cares for certainty? What is the role of proof?

And I shall offer some personal conclusions.

Many of the more sophisticated examples originate in the boundary between mathematical physics and number theory and involve the ζ-function, $\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}$, and its friends [2]. They often rely on the sophisticated use of *Integer Relations Algorithms* — recently ranked among the 'top ten' algorithms of the century [5, 6]. (See [3, 4] and `www.cecm.sfu.ca/projects/IntegerRelations/`.)

As time permits, I shall also describe *WestGrid*, the new Western Canadian computer grid (`www.westgrid.ca`), and my own advanced collaboration facility, *CoLab* (`www.colab.sfu.ca`).

References

1. J.M. Borwein, P.B. Borwein, R. Girgensohn and S. Parnes, "Making Sense of Experimental Mathematics," *Mathematical Intelligencer*, **18**, Number 4 (Fall 1996), 12–18. [CECM 95:032][1]

2. J.M. Borwein and D.M. Bradley, "Empirically Determined Apéry–like Formulae for Zeta(4n+3)," *Experimental Mathematics*, **6** (1997), 181–194. [CECM 96:069]

3. Jonathan M. Borwein and Robert Corless, "Emerging Tools for Experimental Mathematics," *American Mathematical Monthly*, **106** (1999), 889–909. [CECM 98:110]

4. D.H. Bailey and J.M. Borwein, "Experimental Mathematics: Recent Developments and Future Outlook," pp, 51-66 in Volume I of *Mathematics Unlimited — 2001 and Beyond,* B. Engquist and W. Schmid (Eds.), Springer-Verlag, 2000. [CECM Preprint 99:143]

5. J. Dongarra, F. Sullivan, "The top 10 algorithms," *Computing in Science & Engineering,* **2** (2000), 22–23. (See `www.cecm.sfu.ca/personal/jborwein/algorithms.html`.)

6. J.M. Borwein and P.B. Borwein, "Challenges for Mathematical Computing," *Computing in Science & Engineering*, **3** (2001), 48–53. [CECM 00:160].

7. J.M. Borwein, "The Experimental Mathematician: The Pleasure of Discovery and the Role of Proof," to appear in *International Journal of Computers for Mathematical Learning*. [CECM Preprint 02:178]

8. D.H. Bailey, and J.M. Borwein (with the assistance of R. Girgensohn), *Experimentation in Mathematics: Computational Paths to Discovery,* A.K. Peters Ltd, 2003 (in press) ISBN: 1-56881-136-5.

[1] All references except [8] are available at `http://www.cecm.sfu.ca/preprints/`.