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Graduate Talk. **Distances in Hyperbolic Spaces and Negative Definite Kernels.**

Abstract: Real hyperbolic space $H^n_{\mathbf{R}}$, of dimension $n \geq 2$, has a natural metric $d(x, y)$ of negative curvature. It has long been known that $\sqrt{d(x,y)}$ is a Hilbert space distance. In other words, $d(x, y)$ is a *negative definite kernel*. This fact has a geometric explanation, based on an integral formula which expresses $d(x, y)$ in terms of the set of hyperplanes which meet the geodesic segment between x and y.

For complex hyperbolic hyperbolic spaces the distance function is again known to be a negative definite kernel, but a geometric explanation for this is lacking. On the other hand, for quaternionic hyperbolic spaces the distance is not a negative definite kernel. The underlying reason is a deep rigidity theorem from representation theory.

Colloquium I. **Groups acting on affine buildings and their boundaries.**

Abstract: Symmetries of spaces are described by groups. Conversely, it is often useful to study an abstractly defined group by means of the geometry of some space upon which it acts.

Buildings are natural geometries associated with classical groups over p-adic fields. A striking use of geometry was in Serre's explanation of a theorem of Ihara (c.1966) : a discrete torsion free subgroup G of $SL_2(\mathbf{Q}_p)$ is a free group. Serre deduced Ihara's result from the observation that G acts freely upon the *tree* of $SL_2(\mathbf{Q}_p)$.

The *affine buildings* associated with higher rank groups such as $SL_n(\mathbf{Q}_p)$, $n \geq 3$, are higher dimensional analogues of trees, but have a more rigid structure. This talk will introduce, from the viewpoint of an analyst, some ideas of affine buildings and their boundaries.

Colloquium II. **Operator algebras, boundaries of buildings and K-theory.**

Abstract: Spaces that arise in analysis are often pathological and cannot be studied by classical geometric methods. One can sometimes consider instead an associated algebraic object.

Let X be a finite connected graph. The fundamental group Γ of X is a free group and acts on the universal covering tree Δ and on its boundary $\partial\Delta$, endowed with a natural topology. The action of Γ on $\partial\Delta$ is "bad", in the sense that the quotient space $\Gamma\backslash\partial\Delta$ is not Hausdorff. However, this action may be studied by means of the crossed product C^*-algebra $C(\partial\Delta) \rtimes \Gamma$. The structure of this algebra can be explicitly determined. It is a *Cuntz-Krieger algebra*.

Similar algebras may be defined for boundary actions on affine buildings of dimension ≥ 2. These algebras have a structure analogous to that of a simple Cuntz-Krieger algebra and this is the motivation for a theory of higher rank Cuntz-Krieger algebras, which has been developed by T. Steger and G. Robertson. The K-theory of these algebras can be computed explicitly in some cases. Moreover, the class [1] of the identity element in K_0 always has torsion. This talk will outline some of the geometry and algebra involved.