Graduate Talk

A Non-Euclidean Big Bang

We will follow a moving shock wave in a hyperbolic non-euclidean geometry. The Euclidean analog would be a line of particles moving inside an equilateral triangle. When a particle hits an edge of the triangle, it reflects according to the standard rule that the angle of incidence equals the angle of reflection. We will examine the same situation in a non-Euclidean triangle of particular interest in number theory. Here particles move along non-Euclidean straight lines. The reason for the title of this talk will be apparent in the pictures. The cases of both continuous and discrete shock waves (only a finite number of particles) are of interest and are related to current mathematical problems. The talk will include all the necessary background on non-euclidean geometry.

Colloquium I

Graphs and Their Zeta Functions

The Riemann zeta function has had many different generalizations. One of the more recent generalizations is that of the zeta function of a finite graph. Just as the Riemann zeta function may be defined by an infinite product (an "Euler product") over primes, so the zeta function of a graph is given by an infinite product over"prime paths". There is even a Riemann Hypothesis, sometimes true and sometimes not. There are some amazing identities that come out of this. The background graph theory and zeta function material will be included.

Colloquium II

Quadratic Fields in the 20th Century

The class-number of a quadratic field is a measure of how close the integers of the field come to having the unique factorization property. Class-number one is equivalent to unique factorization. Quadratic fields come in two flavors, real and complex. Real quadratic fields are those which are subsets of the real numbers and complex quadratic fields are the rest. Two hundred years ago, Gauss conjectured there are only nine complex quadratic fields with class-number one, and only a finite number of complex quadratic fields with any fixed class-number. One hundred years ago, nothing was known towards these conjectures, although the relevant zeta and L-functions had been introduced and Dirichlet had proved his famous class-number formula in the 1830's. In this talk, we will discuss a century's worth of extraordinary progress, which led to proofs of Gauss's two conjectures as well as new questions, some still unanswered.