Inside-Out: Inverse Boundary Problems

Lecture 1 (graduate): *On Calderón's inverse problem*

Lecture 2 (colloquium): *Electrical Impedance Tomography*

Lecture 3 (colloquium): *Travel Time Tomography*

(Combined) Abstract

In inverse boundary problems one attempts to determine properties of a medium by making measurements at its boundary. In the lectures we will concentrate on two inverse problems, Electrical Impedance Tomography and Travel Time Tomography, which arise in medical imaging, geophysics, and other fields.

In the first lecture we will introduce Calderón's problem which consists of determining the electrical conductivity inside a body by making voltage and current measurements at the boundary. This inverse problem is also called Electrical Impedance Tomography (EIT). The boundary information is encoded in the Dirichlet-to-Neumann (DN) map and the inverse problem is to determine the coefficients of the conductivity equation (an elliptic partial differential equation) knowing the DN map.

In the second lecture we will consider the problem of determining cracks from the DN map as well as EIT for anisotropic conductivities. The latter can be formulated in dimension three or higher as the problem of determining a Riemannian metric from the associated DN map for the Laplace-Beltrami operator.

In the third lecture we will discuss Travel Time Tomography, also called in differential geometry the boundary rigidity problem. In this case the information is encoded in the boundary distance function which measures the lengths of geodesics joining points of the boundary of a compact Riemannian manifold. The inverse problem consists of determining the Riemannian metric from the boundary distance function. This problem arises also in geophysics in an attempt to determine the structure of the interior of the Earth by measuring the travel times of seismic waves going through the Earth.