Nonlinear and adaptive approximation : mathematical foundations and algorithms
Parts I, II, and III

Approximation of an arbitrary function by simpler ones is an important area of mathematical analysis, with applications as diverse as numerical simulation of PDE's, signal and image processing, and statistical learning theory. Classical approaches pick the approximant in a fixed space (polynomials of a certain degree, finite elements, splines). In contrast, adaptive methods consist in letting this space depend on the data to be approximated. These methods are in essence nonlinear and are more powerful, but also more complicated to analyze and implement. These three lectures will aim to present some of the mathematical foundations of nonlinear approximation, as well as some of the state-of-the-art algorithms.

Lecture 1 will introduce some general ideas and motivation, and a basic yet instructive example (piecewise constant approximation) will be studied. It will be accessible to all graduate students (and even undergraduate) with some familiarity with mathematical analysis.

Lecture 2 will focus on approximation properties of wavelets and finite elements, in relation to smoothness spaces. It will be accessible to graduate students who have some familiarity with function spaces.

Lecture 3 will treat a more advanced topic, namely the extraction of an optimal approximation of an arbitrary function by an n-term linear combination from a general dictionary, using so-called greedy algorithms.