Third Talk

Self-interpenetration in Elasticity

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Abstract

Singularities of stress and strain fields are predicted in many problems of elasticity theory; especially at corners, bimaterial interfaces and cracks, and at places near boundaries at which traction and displacement boundary conditions neighbor one another. There are also singularities known to be induced by the presence of zones of converging material symmetry in bodies such as in the case of cylindrical aeolotropy. In such cases of singular or even near singular behavior, it is possible that the theory could predict strange and unacceptable deformation behavior such as the interpenetration of material. If this be the case, then the theory needs to be revised in some way that ultimately respects the condition that allowable deformations are 1-to-1 and onto.

While it has been argued that the classical linear theory of elasticity should not be taken seriously as a predictor of material behavior in the neighborhood of stress singularities, nevertheless the theory, properly constrained to eliminate the possibility of self-interpenetration, has received no attention. This kind of constraint is highly nonlinear, even within the classical linear theory of elasticity, and when it is active it is expected to give rise to the existence of appropriate internal and/or boundary constraint reaction fields.

A constrained variational theory to study such problems is discussed and the effect of applying this approach is exhibited by an example from the classical elasticity literature.