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Talk 1: Graduate talk:

Are polynomials lightweight?

In the 1880's, Weierstrass proved that a continuous function $f : [-1, 1] \to \mathbb{R}$ can be approximated by polynomials uniformly on $[-1, 1]$. This elementary theorem has some easy to state modern ramifications, but are they lightweight? These include:

(I) We can approximate $f(x) = |x|$ by polynomials of degree n with rate $\frac{1}{n}$, but what is the constant?

(II) Why is it that when we approximate $|x|$ by rational functions with numerator, denominator degree n, the rate increases to $e^{-\pi\sqrt{n}}$?

(III) I've lost my taste for ordinary polynomials, only Müntz will do. Is there a cure?

(IV) I feel confined by $[-1, 1]$, why can't I do it on $(-\infty, \infty)$?

(V) What's the best way for a polynomial to develop its potential?

Talk 2 What's ν in Orthogonal Polynomials (or you go your way and I'll go mine)

Let ν be a positive measure on the real line with all finite power moments. Then one may define orthonormal polynomials $\{p_n\}$ satisfying

$$\int_{-\infty}^{\infty} p_n p_m \, d\nu = \delta_{mn}.$$

The behavior of p_n as the degree $n \to \infty$ is an old topic. There are a great many applications, including approximation theory, random matrix theory, combinatorial problems, and integrable systems. In recent years, the introduction of Riemann-Hilbert methods, and the operator theory ideas of Barry Simon, have led to dramatic advances, combined with papers of exponentially growing length and frequency.

We discuss some of the key results, and ideas behind the asymptotics - the Bernstein-Szegö identity, the Riemann-Hilbert formulation, and Simon's steps beyond Szegö. Other orthogonalities and speculations will be offered too.

Talk 3 The Bernstein Constants (or how $|x|$ gets into Acta Math)

In a series of papers from 1913 on, Serge Bernstein investigated polynomial approximation of $f(x) = |x|$ on $[-1, 1]$. Let $E_{n,\infty}[|x|]$ denote the error in approximation of $|x|$ in the uniform norm by polynomials of degree $\leq n$, so that

$$E_{n,\infty}[|x|] = \inf\left\{\||x| - P(x)\|_{L_\infty[-1,1]} : \deg(P) \leq n\right\}.$$

In a long and difficult paper published in 1913 in Acta Math., Bernstein established the limit

$$\lim_{n\to\infty} nE_{n,\infty}[|x|] = \Lambda_1 > 0.$$

Bernstein speculated that $\Lambda_1 = \frac{1}{2\sqrt{\pi}}$, but this was disproved by Carpenter and Varga in 1985. To this day, the value of Λ_1 remains a mystery.

In the late 1930's Bernstein came up with a better method of proof, and showed that for $\alpha > 0$, not an even integer,

$$\lim_{n\to\infty} n^\alpha E_{n,\infty}[|x|^\alpha] = \Lambda_\alpha > 0.$$

Moreover, Λ_α is itself the error in approximation of $|x|^\alpha$ by entire functions of exponential type at most 1:

$$\Lambda_\alpha = \inf\left\{\||x|^\alpha - H(x)\|_{L_\infty(\mathbb{R})} : H \text{ of exponential type } \leq 1\right\}.$$

There is not a single value of α for which Λ_α is explicitly known.

We present old and new results on the determination of Λ_α, and the best approximating entire functions of exponential type, as well as their L_p analogues.