Lecture I: Oppenheim conjecture and related problems

The Oppenheim conjecture, proved in 1986, states that the set of values at integral points of an irrational indefinite quadratic form in $n > 2$ variables is dense in $\mathbf{R}$. I will talk about the history and the proof of the conjecture and will also talk about different problems related to the conjecture.

Lecture II: The normal subgroup theorem

The normal subgroup theorem states that any normal subgroup N of a higher rank irreducible lattice Γ is either central or has finite index. This looks like a purely algebraic result, but the proof, given in the mid-seventies, uses "transcendental" methods such as property (T) of Kazdhan, and the study of "Γ"-invariant algebras of measurable sets on the Furstenberg boundary. One of the ingredients of the proof is a generalization of the classical density point theorem from measure theory. I will also mention some recent developments related to the normal subgroup theorem.

Lecture III: Discrete groups of affine transformations

About 30 years ago, L. Auslander conjectured that if a discrete group H acts properly by affine transformation on $\mathbf{R}^n$ and $\mathbf{R}^n/H$ is compact, then H is virtually polycyclic. I will talk about the current status of the Auslander conjecture. Another topic to be covered is affine proper actions without the assumption that the quotient space is compact.